1. Let $X$ be a topological space. For any subset $A$ of $X$, is it always true that $X \setminus \overline{A} = \text{int}(X \setminus A)$? Prove your assertion. (Here $\overline{A}$ denotes the closure of $A$ and $\text{int}(B)$ denotes the set of interior points of a set $B$.)

2. Let $B = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 1\}$, $p = (1/2, 0)$, and $q = (-1/2, 0)$. Denote $M = B \setminus \{p, q\}$. Is $M$ homotopic to the boundary of $B$? Prove your assertion.

3. Let $X$ be the union of the unit sphere $S^2 \equiv \{(x, y, z) \in \mathbb{R}^3; x^2 + y^2 + z^2 = 1\}$ with the two line segments
   
   \{(0, y, 0); |y| \leq 1\} \cup \{(0, 0, z); |z| \leq 1\}.

   Compute the fundamental group of $X$ based at $(0, 1, 0)$.

4. Let $E$ be a subset of a topological space $Y$. Suppose that $f : Y \to E$ is a continuous map such that $f(x) = x$ for all $x \in E$. Show that if $Y$ is Hausdorff, then $E$ is a closed subset of $Y$.

5. Let $\text{Mat}_2(\mathbb{R})$ be the set of $2 \times 2$ real matrices with the topology obtained by regarding $\text{Mat}_2(\mathbb{R})$ as $\mathbb{R}^4$. Let

   $$\text{SO}(2) = \{A \in \text{Mat}_2(\mathbb{R}); A^T A = I_2, \det A = 1\}$$

   where $A^T$ denotes the transpose of $A$, and $I_2$ is the $2 \times 2$ identity matrix.

   (i) Show that $\text{SO}(2)$ is compact.
   (ii) Is $\text{SO}(2)$ connected? Prove your assertion.

6. Find a simply-connected covering space for the connected sum $\mathbb{R}P^2 \# \mathbb{R}P^2$. Justify your reasoning. (The space $\mathbb{R}P^2$ is the quotient space of the unit sphere $S^2$ obtained by identifying the antipodal points.)