Preliminary Exam **Math 307** Summer, 2008

Name:______________________________ Section:____________

1. Prove or disprove that every compact Hausdorff space is normal.

2. A metric space is called **complete** if every Cauchy sequence converges. The metric is called a complete metric. Prove or disprove the following: Let (X, d) be a metric space. Then there is a complete metric $\bar{d}$ on X so that $(X, \bar{d})$ is homemorphic to (X, d).

3. Let $f : X \to Y$ be a continuous map from a topological space X into another topological space Y. Under what additional conditions, can one prove that f is uniformly continuous? Note that one wants those conditions to be **optimum** to make a clean result without being too trivial. State the definition of being uniformly continuous and an appropriate theorem under the additional conditions, and also prove the theorem.

4. Prove or disprove:

   (a) If A is a nowhere dense subset of X, then $X - A$ is dense. Note that A is said to be **nowhere dense** in X if $\overline{A}$ contains no nonempty open set of X, where $\overline{A}$ is the closure of A in X.

   (b) If $A \subset X$ and $X - A$ is dense in X, then A is a nowhere dense subset of X.

   (c) Suppose A is a compact subset of X. Then $\overline{A}$ is compact.

   (d) Let $f : X \longrightarrow Y$ be a continuous function from a compact space X to a Hausdorff space Y. Then f is a closed function.

5. Let X and Y are homeomorphic topological spaces. Prove or disprove the following.

   (a) Any one-to-one function from X onto Y is a homeomorphism.

   (b) Any continuous one-to-one function from X onto Y is a homeomorphism.

6. If a non-compact topological space X is embedded as a dense subset in a compact topological space X^*, X^* is called a compactification of X. When the complement $X^* - X$ consists of n points, X^* is called an n-point compcactification.

   1) Give an outline for constructing a two point compactification of R^2 with the standard topology.

   2) Prove or disprove that two points compactifications (if there are more than one) of R^2 with the standard topology are unique up to homeomorphisms.