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Topology Preliminary Exam Fall 2002

1. (a) Let $f : X \longrightarrow Y$ be a continuous map. If C is a compact subset of X, prove that $f(C)$ is compact.

(b) If C is a compact subset of X and X is Hausdorff, prove that C is a closed subset of X.

2. Let A be a subset of the topological space X and define $B(A) = \{x \in X | U \cap A \neq \emptyset \neq U \cap (X - A)$ for all open neighborhoods U of $x\}$. Show that A is both open and closed in X if and only if $B(A) = \emptyset$.

3. Let $f : X \longrightarrow Y$ be a continuous map and let $G = \{(x, y) | y = f(x) \text{ and } x \in X\} \subset X \times Y$. Prove that G is homeomorphic to X.

4. Let **T** be the collection of sets $U \subset R^2$ such that U is either the empty set or for each $(x, y) \in U$, there is an open line segment in **each** direction about (x, y) that is contained in U.

a) Show **T** is a topology on R^2.

b) Compare **T** with the standard topology; that is, is it finer, coarser, the same or none of these?

c) Let L denote a straight line in R^2. Compare the subspace topology on L induced by **T** with the subspace topology on L induced by the standard topology on R^2.

d) Let S denote a circle in R^2. Compare the subspace topology on S induced by **T** with the subspace topology on S induced by the standard topology on R^2.

5. Let $\{X_\alpha\}_{\alpha \in J}$ be an indexed family of connected spaces and let $X = \prod_{\alpha \in J} X_\alpha$ be the product space. Prove that X is connected.

6. Let Y denote $S^1 \times S^1 \subset R^2$ with the subspace topology. Let X denote $S^1 \times S^1$ with the quotient topology induced by p where $p : I \times I \longrightarrow S^1 \times S^1$ is defined by $p(x, y) = (\cos(2\pi x), \sin(2\pi y))$ and $I = [0, 1] \subset R$. Prove that X is homeomorphic to Y.