

Name: _____

Math 310
Prelim.

1. State and prove an existence and uniqueness theorem that applies to the equation

$$x' + \sin(x) = t, x(0) = 1$$

2. Prove in a Hilbert space H , that if the operator $A : H \rightarrow H$ is compact, then $x_n \rightarrow x$ weakly implies $Ax_n \rightarrow Ax$ in norm. Prove that if A is compact, then the operator norm of A is attained.
3. Find a Green's function for the differential operator $Ax = x''$ with the boundary conditions $x(0) = 0, x'(1) = 0$. Find the operator norm of the operator

$$Gf(s) = \int_0^1 g(s,t)f(t)dt$$

where g is the Green's function mentioned above.

4. State and prove an existence and uniqueness theorem that applies to the equation

$$x'' + \lambda \cos(x) = 0, x(0) = 0, x'(1) = 0$$

for appropriate conditions on λ .

5. Find $\Delta(1/\sqrt{x^2 + y^2 + z^2})$ in terms of distributional derivatives.