1. Suppose $f(t,y)$ is continuous and $f(t,3) = -2$ for all $t$. If $y(t)$ is a solution to $\frac{dy}{dt} = f(t,y)$ with $y(0) < 3$, can $y(t) \to \infty$ as $t$ increase?

**Sol:** Since $f(t,y)$ is continuous and $f(t,3) = -2$ for all $t$, we must have $f(t,y) < 0$ for $y$ close to 3, i.e. when $y(t)$ gets closer to 3, $y(t)$ is always decreasing. Therefore for a solution with initial data $y(0) < 3$, whenever the solution increase towards 3, it becomes decreasing once it gets closer to 3, therefore can never cross 3 and never approach infinity as $t$ increases.

2. Given $f(t,y) = -y^2 + y + 2yt^2 + 2t - t^2 - t^4$. It is known both $y_1(t) = t^2$ and $y_2(t) = t^2 + 1$ are solutions to $\frac{dy}{dt} = f(t,y)$. If $y(t)$ is a solution of $\frac{dy}{dt} = f(t,y)$ with $y(0) = 1/2$, show $t^2 < y(t) < t^2 + 1$ for all $t$.

**Sol:** Since $f(t,y)$ and $f_y(t,y)$ are both continuous on $\mathbb{R}^2$, by uniqueness theorem, two different solutions of $\frac{dy}{dt} = f(t,y)$ can never intersect. In particular, if $y(0) = 1/2$ lies between $y_1(0) = 0$ and $y_2(0) = 1$, so must $y(t)$ lies between $y_1(t) = t^2$ and $y_2(t) = t^2 + 1$ i.e. $t^2 < y(t) < t^2 + 1$ for all $t$. 