Suppose a species of fish in a lake has population modeled by the logistic population model with growth rate $k$, carrying capacity $N$, $t$ measured in years. Adjust the model to account for the following situations.

**a.** (3 pts) 140 fishes harvested each year.

**b.** (3 pts) $\frac{1}{3}$ of the fish harvested annually.

**c.** (4 pts) If $k = 0.3$, $N = 2400$ and there are 3000 fishes in the lake when $t = 0$, what does model in situation b predict in the long run?

**Sol:**

**a.**

$$\frac{dP}{dt} = k \left(1 - \frac{P}{N}\right)P - 140$$

**b.**

$$\frac{dP}{dt} = k \left(1 - \frac{P}{N}\right)P - \frac{1}{3}P$$

**c.**

$$\frac{dP}{dt} = k \left(1 - \frac{P}{N}\right)P - \frac{1}{3}P$$

$$= 0.3 \left(1 - \frac{P}{2400}\right)P - \frac{1}{3}P$$

$$= P \left(0.3 - \frac{1}{3} - \frac{P}{2400}\right)$$

$$= P \left(-\frac{1}{30} - \frac{P}{2400}\right) < 0$$

and $\frac{dP}{dt} \to 0$ as $P$ approaches zero. Therefore in the long run, the population of the fish keep decreasing, eventually goes to zero, i.e. the fish goes distinct in the long run.