Suppose a species of fish in a lake has population modeled by the logistic population model with growth rate \( k \), carrying capacity \( N \), \( t \) measured in years. Adjust the model to account for the following situations.

a. (4 pts) 150 fishes harvested each year.

b. (4 pts) \( \frac{1}{3} \) of the fish harvested annually.

c. (4 pts) number of fish harvested each year proportional to square root of the number of fish in the lake.

d. (8 pts) If \( k = 0.3 \), \( N = 2400 \) and there are 2400 fishes in the lake when \( t = 0 \), what does model in situation b predict in the long run?

Solution: Quantities involved: \( t \) -- time, measured in years.
\[ P(t) \] -- Number of fishes at year \( t \)
\( N \) -- Capacity
\( k \) -- growth rate coefficient.

Since \( t \) is measured in units of years. \( \frac{dP}{dt} \) represents change in the number of fishes per year, therefore

\[ kP\left(1 - \frac{P}{N}\right) \]

represents fish growth per year while number of fishes harvested represents fish decrease per year.

a. \[
\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - 150
\]

b. \[
\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - \frac{1}{3}P
\]

c. \[
\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - a\sqrt{P}
\]

here \( a \) represents the proportionality constant.
d. With the given data, we have

\[
\frac{dP}{dt} = 0.3P(1 - \frac{P}{2400}) - \frac{1}{3}P \\
= -\left(\frac{1}{3} - 0.3\right) P - \frac{0.3}{2400}P^2 \\
< 0
\]

as long as \( P > 0 \). Thus the model predicts the number of fishes keep decreasing, eventually to extinction. (Note the decreasing rate at the starting point is \( \frac{dP}{dt} = 0.3 \times 2400(1 - \frac{2400}{2400}) - \frac{1}{3} \times 2400 = -800 < 0 \), the decreasing rate is getting smaller and smaller as \( P \) approaches 0)