(3) (Game of Fifteen) Two players alternately call out a digit between 1 and 9, inclusively, with no digit being called twice. The winner is the first player to have called out three digits which add up to exactly 15.

(4) (The Game of Thirty) A pile contains thirty pebbles. Two players alternate turns, each removing either one or two pebbles from the pile per turn. The player who removes the last pebble wins. Is it better to go first or second?

(5) (Cartesian Chase) Cartesian Chase is a game played on a rectangular grid with a fixed n
The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy. This question was on the 1995 Putnam Exam.

(11) (.777...) Is there a sure win or higher winning chance strategy?

There are six groups of sticks:

1  Group 1 (1 stick)
11 Group 2 (2 sticks)
111 Group 3 (3 sticks)
1111 Group 4 (4 sticks)
11111 Group 5 (5 sticks)
111111 Group 6 (6 sticks)

Rules:
(a) 2 players take turn to strike off any number of sticks from any group.
(b) To strike off 2 or more sticks in a group, the sticks must be side by side.
(c) If striking off one or some of the sticks from in between a group, then the remaining sticks on the right and on the left are now consider 2 distinct groups.
(d) In order to win, you must leave one last stick for your opponent to strike off.

(12) (Percolation Project and The Game of Hex)

Percolation theory is a relatively new branch of mathematics that deals with how one substance moves through another. The classic example is boiling water percolating through coffee grounds. We will introduce the subject by analyzing some games.

The game of Hex is played by two persons or two pairs of partners. The players select sides on the board and different colored playing pieces. The players alternate turns by placing one of their pieces on any unoccupied hexagon. The object of the game is for one of the players (the winner) to complete an unbroken chain of pieces between his two sides. The chain may twist and turn freely but must be unbroken. The corners belong to both sides.

Play the game of Hex two to four times, taking turns making the first move. Next play the game on an 8 x 8 checkerboard of squares. Answer these questions for each of the two games.
(a) Can both players win?
(b) Can both players lose?
(c) Must at least one player win?
(d) Can the player who goes first always win? If so, describe a winning strategy.

Now try a different game on the Hex board. The first player puts a piece on the center hexagon. The second player may place two markers anywhere she likes. The first player then gets to place one marker, the second player two, and so on. The object of the game is for the first player to complete an unbroken chain to a hexagon on the boundary. The second player's job is to prevent this.
(a) Can either player always win? If so, describe a winning strategy.
(b) Discuss any visual images that came to you while investigating these percolation games.