Math 2142 Homework 11: Due Friday April 24

Problem 1. Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors in $V_n$ and let $c \in \mathbb{R}$ be a scalar. Prove that
\[
\begin{align*}
    c(\mathbf{u} + \mathbf{v}) &= c\mathbf{u} + c\mathbf{v} \\
    \mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) &= \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} \\
    ||c\mathbf{v}|| &= |c| ||\mathbf{v}||
\end{align*}
\]

Problem 2. Do Problems 19 and 20 in Exercises 12.8 in the textbook.

Problem 3. Do Problems 2, 3, 7, 9 and 10 in Exercises 12.15 in the textbook.

Problem 4. Let $\mathbf{v}$ be a vector in $V_n$. Prove that $S = \{\mathbf{v}\}$ is linearly independent if and only if $\mathbf{v} \neq \mathbf{0}$.

Problem 5. Let $\mathbf{u}$ and $\mathbf{v}$ be nonzero vectors in $V_n$. Prove that $S = \{\mathbf{u}, \mathbf{v}\}$ is linearly independent if and only if $\mathbf{u} \neq c\mathbf{v}$ for any scalar $c \in \mathbb{R}$.

Problem 6. Determine whether each of the following sets of vectors is a basis for $V_3$.
\[
S = \{\langle 2, 0, 3 \rangle, \langle 5, 4, -2 \rangle, \langle 2, -1, 1 \rangle\} \\
T = \{\langle 1, 2, -3 \rangle, \langle 3, 1, -2 \rangle, \langle 5, -5, 6 \rangle\}
\]

\textit{Hint.} Since $S$ and $T$ contain 3 vectors, it suffices to check whether they are linearly independent.

Problem 7. Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ be a linearly independent set of vectors in $V_n$ and let $\mathbf{w}$ be a vector in $V_n$ such that $\mathbf{w} \not\in \text{Span}(S)$. Prove that $S \cup \{\mathbf{w}\}$ is linearly independent.

\textit{Hint.} Do this problem by contradiction. Assume that $S \cup \{\mathbf{w}\}$ is linearly dependent and then show that $\mathbf{w}$ is in $\text{Span}(S)$. 