Math 2142 Homework 10: Due Friday April 17

Problem 1. In Exercises 12.4 of the textbook, do Problems 1, 2, 5, 6 and 7.

Hint for Problem 5. In Problem 5, you are given vectors \( A = \langle 2, 1 \rangle \) and \( B = \langle 1, 3 \rangle \). Therefore, for any \( x, y \in \mathbb{R} \),

\[
xA + yB = x\langle 2, 1 \rangle + y\langle 1, 3 \rangle = \langle 2x + y, x + 3y \rangle
\]

You are given an arbitrary vector \( C = \langle c_1, c_2 \rangle \) and asked to find \( x \) and \( y \) such that \( C = xA+yB \). That is, you need to find \( x \) and \( y \) so that

\[
\langle c_1, c_2 \rangle = \langle 2x + y, x + 3y \rangle
\]

Writing this out as components, you get the following system of equations

\[
\begin{align*}
c_1 &= 2x + y \\
c_2 &= x + 3y
\end{align*}
\]

To find \( x \) and \( y \), you need to solve this system of equations. Your answer will express \( x \) and \( y \) in terms of \( c_1 \) and \( c_2 \).

Hint for Problem 6(b). For 6(b), you need to show that the only solution to

\[
\langle 0, 0, 0 \rangle = x\langle 1, 1, 1 \rangle + y\langle 0, 1, 1 \rangle + z\langle 1, 1, 0 \rangle
\]

if \( x = y = z = 0 \). To find find \( x, y \) and \( z \) solving this equation, combine the vectors on the right side to get

\[
\langle 0, 0, 0 \rangle = \langle x + z, x + y + z, x + y \rangle
\]

and separate into components

\[
\begin{align*}
0 &= x + z \\
0 &= x + y + z \\
0 &= x + y
\end{align*}
\]

Solve this system of equations to see that the only solution is \( x = y = z = 0 \).

Hint for 7(b). The set-up for 7(b) is just like 6(b) except with a different set of vectors. Follow the same pattern. When you solve the system of equations, you should fine that there are many solutions, not just \( x = y = z = 0 \). Any of these solutions will make \( xA + yB + zC = \vec{0} \) so you can just pick one.

Hint for 7(c). For 7(c), you will follow the pattern of 7(b) except you use \( \langle 1, 2, 3 \rangle \) instead of \( \langle 0, 0, 0 \rangle \). Using the components, set up a system of equations and try to solve them. This time, you should find that there is no solution!
Problem 2. In Exercises 12.8 of the textbook, do Problems 1, 5, 6, 9 and 15.

Hint for Problem 5. Let \( C = \langle x, y, z \rangle \). You want \( C \) to satisfy \( A \cdot C = 0 \) and \( B \cdot C = 0 \). You have

\[
A \cdot C = \langle 2, 1, -1 \rangle \cdot \langle x, y, z \rangle = 2x + y - z
\]

so you know you need \( 2x + y - z = 0 \). Write out \( B \cdot C = 0 \) and you will get another equation with \( x, y \) and \( z \). Solve this system of equations to find values of \( x, y \) and \( z \) giving a vector \( C \) with the desired property.

Problem 3. In Exercises 12.11 of the textbook, do Problems 1 and 5.

Hint for Problem 5. You are given the three vertices of the triangle in \( \mathbb{R}^3 \). Use these points to find the vectors representing the sides of the triangle. Once you have the vectors for the sides of the triangle, you can find the angles between them.