Problem 1. Prove that if $k$ is a prime number, then $\sqrt{k}$ is irrational.

*Hint.* I would do Problem 1 by contradiction following the example from class when $k = 2$. You can use the fact that if $k$ is prime and $k$ divides $ab$, then $k$ must divide at least one of $a$ or $b$.

Problem 2. Let $r < 0$ be a negative real number. Use the Archimedean property (or the consequences of the Archimedean property we proved in class) to prove that there is an $n \in \mathbb{N}^+$ such that $r < -1/n < 0$.

Problem 3. Prove that if $u$ is a rational number and $v$ is an irrational number, then $u + v$ is an irrational number.

*Hint.* I would do Problem 3 by contradiction. Assume that $u$ is rational (so you can write $u = p/q$) and $v$ is irrational. For a contradiction, assume that $u + v$ is rational (and thus you can write $u + v = a/b$). To obtain a contradiction, try to show that $v$ is actually rational. Start with the equation $u + v = a/b$ and rewrite it until you have shown that $v$ is equal to a fraction of integers.

Problem 4. Let $A \subseteq \mathbb{R}^+$ be a subset of the positive real numbers which is bounded above and let $r > 0$ be a positive real number. Define the following two subsets of $\mathbb{R}$:

$$B = \{r + a \mid a \in A\} \quad \text{and} \quad C = \{ra \mid a \in A\}$$

Prove that if $\sup(A) = u$, then $\sup(B) = r + u$ and $\sup(C) = ru$.

*Hint.* For the case of $\sup(B) = r + u$, you need to show two things. First, show that $r + u$ is an upper bound for the set $B$. That is, show that if $b \in B$, then $b \leq r + u$. Second, show that $r + u$ is the *least* upper bound of $B$. That is, show that if $\hat{r} < r + u$, then $\hat{r}$ is not an upper bound for $B$. 