We prove that Conjecture 1.1 and 1.2 hold true for cluster algebras of Dynkin type. For definitions and notations we refer to [ASS2].

**Theorem 0.1.** Let $\mathcal{A}$ be a cluster algebra of Dynkin type, then $\mathcal{A}$ is unistructural.

**Proof.** Assume that $\mathcal{A}$ is given two cluster structures $\mathcal{X} = \cup x_\alpha = \cup x'_\beta$ where $x_\alpha$ and $x'_\beta$ are clusters in their respective structures. Denote the two cluster structures by $S$ and $S'$, respectively. Let $x = \{x_1, x_2, \ldots, x_n\}$ be the initial cluster of $S$. We claim that $x$ is also a cluster in $S'$. If not, then there exist two initial variables $x_i, x_j$ which are not compatible in $S'$: indeed, this can be seen by using the well-known bijections between clusters and tilting objects in the associated cluster category, and between cluster variables and rigid indecomposable objects. Because of the positivity theorem, each of $x_i$ and $x_j$ is a positive element in $\mathcal{A}$ (in both structures), hence so is their product $x_i x_j$. Cerulli-Irelli showed that the cluster monomials form an atomic basis for $\mathcal{A}$, see [C] Th.1.1, which implies that every positive element is a linear combination of cluster monomials with non-negative coefficients.

Therefore the product $x_i x_j$ in the structure $S'$ can be written as a positive linear combination of cluster monomials: $x_i x_j = \sum \lambda_{M'} M'$. Each of the cluster monomials $M'$ is a product of $S'$-compatible cluster variables and each of these cluster variables can be written as a positive Laurent polynomial in $x_1, \ldots, x_n$, because the latter is a cluster of $S$ and both structures have the same set of cluster variables. Thus the cluster monomial $M'$ can also be written as a positive Laurent polynomial $L(M')$ in $\{x_1, \ldots, x_n\}$.

Replacing each $M'$ by $L(M')$ in the sum $\sum \lambda_{M'} M'$, we get

$$x_i x_j = \sum \lambda_{M'} L(M')$$

and because of positivity there is no cancellation of terms in the right hand side. Therefore the sum $\sum \lambda_{M'} M'$ has only one term $M' = x_i x_j$ and $\lambda_{M'} = 1$. But this means that $x_i$ and $x_j$ are $S'$-compatible, a contradiction. This proves that $\{x_1, \ldots, x_n\}$ is a cluster in the structure $S'$.

In order to complete the proof it suffices to show that the quiver $Q'$ of the cluster $\{x_1, \ldots, x_n\}$ in the structure $S'$ is equal or opposite to the quiver $Q$ of the same cluster in the structure $S$. The mutations $\mu_i$ and $\mu'_i$ in the direction $i$ applied to the cluster $\{x_1, \ldots, x_n\}$ in both structures $S$ and $S'$ will produce a variable whose denominator is $x_i$. Namely,

$$\mu_i(x_i) = \frac{\prod_{i \rightarrow j \in Q} x_j + \prod_{i \leftarrow j \in Q} x_j}{x_i} \quad \text{and} \quad \mu'_i(x_i) = \frac{\prod_{i \rightarrow j \in Q'} x_j + \prod_{i \leftarrow j \in Q'} x_j}{x_i}.$$
\( \mu'_i(x_i) \) and therefore either
\[
\prod_{i \to j \text{ in } Q} x_j = \prod_{i \to j \text{ in } Q'} x_j \quad \text{and} \quad \prod_{i \to j \text{ in } Q} x_j = \prod_{i \to j \text{ in } Q'} x_j
\]
or
\[
\prod_{i \to j \text{ in } Q} x_j = \prod_{i \to j \text{ in } Q'} x_j \quad \text{and} \quad \prod_{i \to j \text{ in } Q} x_j = \prod_{i \to j \text{ in } Q'} x_j.
\]
Since \( i \) is arbitrary and \( Q \) is connected, this implies that \( Q = Q' \) or \( Q = Q^{\text{op}} \). □

**Corollary 0.2.** Conjecture 1.1 holds true for cluster algebras of Dynkin type.

**Proof.** This follows from the above theorem and Theorem 1.4. □

**References**

[ASS2] I. Assem, R. Schiffler and V. Shramchenko, Cluster automorphisms and compatibility of cluster variables, Glasgow Mathematical Journal

[C] G. Cerulli Irelli, Positivity in skew-symmetric cluster algebras of finite type, arxiv: 1102.3050