Name:_____________________

Math 210
Final Exam

1. Find the velocity, acceleration, and tangential and normal components of acceleration for the trajectory

\[ \vec{R}(t) = (t^3)\vec{i} + 2\sin(t^3)\vec{j} + 2\cos(t^3)\vec{k} \]
2. Let \( f(x, y) = \frac{\sqrt{x}}{y} \).

(a) Write the equation of the tangent plane to the surface determined by \( f \) at the point \((1, 1, 1)\).

(b) Use linear approximation to find \( \frac{\sqrt{0.98}}{1.02} \)
3. Let $f(x, y) = (x - 1)^2 + (y - 1)^2$.
   
   (a) Find the critical points of $f$ and classify them as maxima, minima, or saddle points.
   
   (b) Find the absolute extrema of $f$ over the triangle with vertices (6, 0), (0, 6), (6, 6)
4. Let \( f(x, y, z) = x^2 - y^2 + 2z^2 \) and let \( P = (-1, 1, 1) \).

(a) Calculate the derivative of \( f \) in the direction of the vector \( \vec{v} = \vec{i} - 2\vec{j} + 2\vec{k} \).

(b) Find the direction of steepest increase in \( f \) and find the rate of that increase.
5. (a) Evaluate the integral \( \int \int_D x \, dA \) where \( D \) is the triangle with vertices \((0, 0), (3, 0), \) and \((0, 3)\)

(b) Evaluate the integral \( \int_0^1 \int_{3y}^3 e^{x^2} \, dx \, dy \)
6. Evaluate the integral \( \int \int_L f_V(x^2 + y^2 + z^2) \, dV \) where \( V \) is bounded below by \( z = -2 \) and above by \( z = -\sqrt{x^2 + y^2} \).
7. Let \( \vec{F}(x, y) = (x)\vec{i} + (x y)\vec{j} \) and let \( C \) be the line segment from \((0,0)\) to \((2,3)\). Compute \( \int_C \vec{F} \cdot d\vec{r} \).
8. Let \( \vec{F}(x, y) = (x - y^2 \sin(x)) \vec{i} + (2y \cos(x) + y) \vec{j} \) and let \( C \) be the line segment from \((1, 1)\) to \((2, 3)\).

(a) Compute \( \int_C \vec{F} \cdot d\vec{r} \).

(b) Suppose instead that we take the path \( \vec{r}(t) = <1 + t^2, 1 + 2t^2> \) from \((1, 1)\) to \((2, 3)\). How does the value of the line integral in (a) change. Explain your answer fully. Do not evaluate the integral.
9. Calculate the integral $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = x\vec{i} + (y + 1)\vec{j}$ and $S$ is the part of the surface $x + y + z = 1$ in the first octant.
10. Calculate the integral \( \int \int_S \vec{F} \cdot d\vec{S} \) where \( \vec{F} = x^2\hat{i} + (y + 1)\hat{j} \) and \( S \) is the unit cube in the first octant.