You must show your work to receive credit.

1. Consider the linear system $\vec{Y}' = A\vec{Y}$ where

   $$A = \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix}$$

   (a) Compute the eigenvalues of $A$ and find the general solution. Sketch the phase plane.

   (b) Find the solution for the initial value problem $Y_1(0) = 1, Y_2(0) = 0.$
2. Consider the linear system $\ddot{\mathbf{Y}} = A\dot{\mathbf{Y}}$ where

$$A = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}$$

(a) Compute the eigenvalues of $A$ and find the general solution.

(b) Sketch the phase plane.

(c) Find the solution for the initial value problem $Y_1(0) = 1, Y_2(0) = 0$. 
3. Find the solution for the problem $y'' + 4y' + 5y = 0, y(0) = 1, y'(0) = 2$
4. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

\[
\frac{dx}{dt} = x(2 - x - y) \\
\frac{dy}{dt} = y(3 - x - 3y).
\]

Draw the curves \(x(t)\) and \(y(t)\) if \(x(0) = 1\) and \(y(0) = 1\)