1. (4 pts) Determine whether the following series is **absolutely** or **conditionally** convergent.

\[ \sum_{n=0}^{\infty} (-1)^n \frac{1}{n^2 + 3}. \]

**Solution:**

First notice that the series is convergent since

\[ \lim_{n \to \infty} \frac{1}{n^2 + 3} = 0, \]

and since the derivative of \( f(x) = \frac{1}{x^2 + 3} \) is \( f'(x) = \frac{-2x}{(x^2 + 3)^2} \), which is negative, hence the function \( f \) is decreasing. Thus \( b_n = \frac{1}{n^2 + 3} \) is decreasing. Thus by the Alternating series test we get that

\[ \sum_{n=0}^{\infty} (-1)^n \frac{1}{n^2 + 3} \]

is convergent. Now to check absolute convergence we must check to see if

\[ \sum_{n=0}^{\infty} \left| (-1)^n \frac{1}{n^2 + 3} \right| = \sum_{n=0}^{\infty} \frac{1}{n^2 + 3} \]

is convergent. Taking \( a_n = \frac{1}{n^2 + 3} \) and \( b_n = \frac{1}{n^2} \), we see that

\[ a_n = \frac{1}{n^2 + 3} < \frac{1}{n^2} = b_n \]

for all \( n \). But since \( \sum \frac{1}{n^2} \) is convergent, then

\[ \sum_{n=0}^{\infty} \frac{1}{n^2 + 3} \]

is convergent by the Comparison test. Thus since both

\[ \sum_{n=0}^{\infty} (-1)^n \frac{1}{n^2 + 3} \quad \text{and} \quad \sum_{n=0}^{\infty} \left| (-1)^n \frac{1}{n^2 + 3} \right| \]

are convergent, then the series is **absolutely convergent**.
2. (3 pts) Use the **divergence** test on the following series. *State your conclusion.*

\[ \sum_{n=0}^{\infty} \frac{3n^2}{4n^2 + 1} \]

**Solution:**
To use the divergence test we simply take the following limit:

\[ \lim_{n \to \infty} \frac{3n^2}{4n^2 + 1} = \lim_{n \to \infty} \frac{3n^2}{n^2} \frac{4}{n^2} + \frac{1}{n^2} \]

\[ = \lim_{n \to \infty} \frac{3}{4 + \frac{1}{n^2}} \]
\[ = \frac{3}{4 + 0} = \frac{3}{4}. \]

But since this is NOT zero, then we know that the series must **diverge** by the Divergence Test.

3. (3 pts) Write out the first three *nonzero* terms of the following series.

\[ \sum_{n=3}^{\infty} \frac{n-3}{n} x^n \]

**Solution:**
Notice the series starts with \( n = 3 \). A table might help:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( a_n = \frac{n-3}{n} x^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( \frac{2}{3} x^3 = 0 )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{1}{4} x^4 = \frac{1}{4} x^4 )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{2}{5} x^5 = \frac{2}{5} x^5 )</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{2}{6} x^6 = \frac{1}{3} x^6 )</td>
</tr>
</tbody>
</table>

Therefore the first nonzero terms of the series are

\[ \sum_{n=0}^{\infty} \frac{n-3}{n} x^n \approx \frac{1}{4} x^4 + \frac{2}{5} x^5 + \frac{1}{2} x^6. \]