Using separation of variables to solve the initial value problem
\[ \frac{dy}{dt} = t^2 y^3, \quad y(0) = -1. \]
(Be careful when you choose a square root: use the initial value to make the choice)

Solution: By separation of variables,
\[ \frac{dy}{y^3} = t^2 dt, \]
\[ -\frac{1}{2y^2} = \frac{1}{3} t^3 + c_1, \]
\[ y^2 = \frac{1}{2(1/3 \cdot t^3 + c_1)}, \]
\[ y^2 = -\frac{3}{2t^3 + c_2}, \quad \text{where } c_2 = 6c_1. \]
The initial condition \( y(0) = -1 \) implies that \( 1 = y(0)^2 = -3/c_2 \), whence we have \( c_2 = -3 \). Therefore
\[ y(t) = -\sqrt{\frac{3}{3 - 2t^3}}. \]
Note that we take the negative branch of the square root, again from the initial condition \( y(0) = -1 < 0 \).