MATH2410 QUIZ 11, DEC 1, 2015

Name: ID: Score:

Find the inverse Laplace transform of \( \frac{4}{s(s + 3)} \).

**Solution** Write \( \frac{4}{s(s + 3)} = \frac{A}{s} + \frac{B}{s + 3} \).

So \( 4 = A(s + 3) + Bs. \) Setting \( s = 0 \) gives \( 3A = 4 \Rightarrow A = 4/3. \) Looking at the \( s \)-coefficients gives \( B = -4/3. \) So

\[
\mathcal{L}^{-1}\left[ \frac{4}{s(s + 3)} \right] = \frac{4}{3} \mathcal{L}^{-1}\left[ \frac{1}{s} \right] - \mathcal{L}^{-1}\left[ \frac{1}{s + 3} \right] = \frac{4}{3}(1 - e^{-3t}).
\]

Solve the given initial-value problem.

\[
\frac{dy}{dt} + 9y = u_5(t), \quad y(0) = -2.
\]

**Solution** Take the Laplace transform gives

\[
\mathcal{L}\left[ \frac{dy}{dt} \right] + 9\mathcal{L}[y] = \mathcal{L}[u_5(t)]
\]

\[
s\mathcal{L}[y] + 2 + 9\mathcal{L}[y] = \frac{e^{-5s}}{s}.
\]

\[
(s + 9)\mathcal{L}[y] = -2 + \frac{e^{-5s}}{s}.
\]

\[
\mathcal{L}[y] = -\frac{2}{s + 9} + \frac{e^{-5s}}{s(s + 9)}.
\]

Note that \( \frac{1}{s(s+9)} = \frac{1}{9} \cdot \frac{1}{s} - \frac{1}{9} \cdot \frac{1}{s+9}. \) So

\[
y = -\mathcal{L}^{-1}\left[ \frac{2}{s + 9} \right] + \frac{1}{9} \mathcal{L}^{-1}\left[ \frac{e^{-5s}}{s} \right] - \frac{1}{9} \mathcal{L}^{-1}\left[ \frac{e^{-5s}}{s + 9} \right]
\]

\[
= -2e^{9t} + \frac{1}{9} u_5(t) - \frac{1}{9} u_5(t) e^{-9(t-5)}
\]

\[
= -2e^{9t} + \frac{1}{9} u_5(t)(1 - e^{-9(t-5)}).
\]