MATH2410 QUIZ 10, DEC 1, 2015

Name: ID: Score:

Find a particular solution to the following differential equation
\[
\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = \cos t.
\]

Solution We complexify the equation to be \( \frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = e^{it} \). Guess \( y_c(t) = Ce^{it} \).
Then we have
\[
-C^{it} + 6iC^{it} + 8C^{it} = e^{it}.
\]
So we have \( C = \frac{1}{1+6i} = \frac{7-6i}{85} \). So
\[
y_c(t) = \frac{7-6i}{85} e^{it} = \frac{7-6i}{85} \left( \cos t + i \sin t \right) = \frac{(7 \cos t + 6 \sin t) + i(7 \sin t - 6 \cos t)}{85}.
\]
So the particular solution to the original equation is the real part of \( y_c(t) \), that is
\[
y_p(t) = \frac{7 \cos t + 6 \sin t}{85}.
\]

For the following equation, determine the frequency of the beats and the frequency of the rapid oscillations, and use the information of the previous two parts, give a rough sketch of a typical solution.
\[
\frac{d^2y}{dt^2} + 4y = \cos \frac{9t}{4}.
\]

Solution The characteristic polynomial of the unforced equation is \( s^2 + 4 \), with roots \( \pm 2i \). So the natural frequency is \( 2/(2\pi) \), and the forcing frequency is \( 9/4/2\pi = 9/(8\pi) \).

The frequency of the beating is
\[
\frac{\frac{9}{4} - 2}{4\pi} = \frac{1}{16\pi}.
\]
The period of one beat is \( 16\pi \approx 50 \).

The frequency of the rapid oscillation is
\[
\frac{\frac{9}{4} + 2}{4\pi} = \frac{17}{16\pi}.
\]

There are 17 rapid oscillations in each beat.