Assignment 1

1. (10 points)
   Use Gaussian elimination with backward substitution to solve the following linear systems.
   
   \[
   \begin{align*}
   4x_1 - x_2 + x_3 &= 8 \\
   2x_1 + 5x_2 + 2x_3 &= 3 \\
   x_1 + 2x_2 + 4x_3 &= 11,
   \end{align*}
   \]
   \[
   \begin{align*}
   4x_1 + x_2 + 2x_3 &= 9 \\
   2x_1 + 4x_2 - x_3 &= -5 \\
   x_1 + x_2 - x_3 &= -9,
   \end{align*}
   \]

2. (10 points) Solve the above linear systems using Gaussian elimination with backward substitution and two-digit rounding arithmetics.

3. (10 points)
   Given the linear system
   \[
   \begin{align*}
   x_1 - x_2 + \alpha x_3 &= -2 \\
   -x_1 + 2x_2 - \alpha x_3 &= 3 \\
   \alpha x_1 + x_2 + x_3 &= 2.
   \end{align*}
   \]
   (a) Find value or values of \(\alpha\) for which the system has no solution.
   (b) Find value or values of \(\alpha\) for which the system has infinite number of solutions.
   (c) Find value or values of \(\alpha\) for which the system has unique solution.

4. (10 points)
   Write a Matlab function
   
   \[
   x = \text{my_low_triag_solve}(L,b)
   \]
   
   that solves the linear system \(Lx = b\), where \(L\) is lower triangular matrix, by the backward substitution. The function’s input an \(n\)-by-\(n\) lower triangular matrix \(L\) and an \(n\)-vector \(b\). The function’s output is a solution vector \(x\) (if such exists).