MATH 1131 – Calculus I

Practice Exam 3

No calculators. Show your work. Clearly mark each answer.

1. Find the linear approximation of \( \sqrt{x} \) at point \( a = 16 \) and using it estimate \( \sqrt{16.04} \).

2. Evaluate the following limits:
   
   (a) \[ \lim_{x \to 0} \frac{\tan(3x)}{\sin(2x)} \]

   (b) \[ \lim_{x \to \infty} \frac{e^{x^2}}{x^2} \]

   (c) \[ \lim_{x \to \infty} \left(1 + \frac{1}{2x}\right)^x \]

3. Find the minimum value of \( 5a + 3b \) for \( a > 0 \) and \( b > 0 \), such that \( ab = 25 \).

4. Using the following graphing guidelines sketch the function \( f(x) = x \ln^2 x \):
   
   (a) What’s the domain of \( f(x) \)?

   (b) Is \( f(x) \) even or odd?

   (c) Find \( f'(x) \).

   (d) Find \( f''(x) \).

   (e) Find the critical points (i.e. where \( f'(x) = 0 \)).

   (f) Find inflection points (i.e. where \( f''(x) = 0 \)).

   (g) Find the intervals on which the function is increasing, decreasing.

   (h) Find the intervals on which the function is concave up, concave down.

   (i) Identify extreme points.

   (j) Locate vertical asymptotes

   (k) Locate horizontal asymptotes

   (l) Find \( x \)-intercepts (i.e. where \( f(x) = 0 \)).

   (m) Find \( y \)-intercept (i.e. when \( f(0) \)).

   (n) Sketch the graph

5. Find the interval on which Rolle’s Theorem applied to \( f(x) = x^2(x+3) \). Find a point \( c \) in that interval at which \( f'(c) = 0 \).