MATH 1050QC
Mathematical Modeling in the Environment
Lecture 15. Basic Plume Model.

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"Puff" and "Plume"

Figure 3.15 from C. Hadlock's book

Momentary Puff Release

Typical Plume Release
Gaussian Plume Model

\[ C = \frac{Q}{2\pi \sigma_y \sigma_z u} e^{-\frac{y^2}{2\sigma_y^2}} \left( e^{-\frac{(z-H)^2}{2\sigma_z^2}} + e^{-\frac{(z+H)^2}{2\sigma_z^2}} \right) \]

where

- \( Q \) — mass of pollutants emerging from the stacks per unit time
- \( u \) — wind velocity
- \( y \) — horizontal coordinate measured perpendicular to wind direction
- \( \sigma_y \) — dispersion coefficient (standard deviation) in the \( y \)-direction (depends on \( x \)).
- \( z \) — elevation above the ground
- \( \sigma_z \) — dispersion coefficient (standard deviation) in the \( z \)-direction (depends on \( x \)).
- \( H \) — effective stack height
The above equation we can rewrite as

\[
C' = \left(\frac{Q}{u}\right) \times \left(\frac{1}{\sqrt{2\pi\sigma_y}} e^{-\frac{y^2}{2\sigma_y^2}}\right) \times \left(\frac{1}{\sqrt{2\pi\sigma_z}} \left(e^{-\frac{(z-H)^2}{2\sigma_z^2}} + e^{-\frac{(z+H)^2}{2\sigma_z^2}}\right)\right)
\]

source strength \hspace{2cm} \text{diffusion effect in } y \text{ direction} \hspace{2cm} \text{diffusion effect in } z \text{ direction}

Thus the concentration \( C' \) is the product of three quantities described above.
Natural Questions

1. Why does not $x$ show up in the equation? Should not the concentration of pollutant depend on the distance $x$ from the source of release?

2. How would one calculate the dispersion coefficients $\sigma_y$ and $\sigma_z$?

3. Where does the plume equation come from?
Horizontal dispersion coefficient $\sigma_y$ as a function of downwind distance for each stability class
Horizontal dispersion coefficient $\sigma_z$ as a function of downwind distance for each stability class.
3D representation of plume coordinate system and dispersion effect in horizontal and vertical direction

figure 3.18 from C. Hadlock’s book
You live two miles due east of a coal-fired utility power plant that produces electricity for your city. The stack on the plant is 350 feet high, and the ground is level. On a given day, the sun is shining brightly and the wind is blowing from southwest to northeast at 10 mph.

Measurements at the plan stack of the concentration of nitrogen oxides in the exhaust gas show that such pollutant are released at the rate of 80 pounds per minute. What would you expect the concentration of nitrogen oxides to be at your residence?
Problem geometry

Note: be sure to convert coordinates to units consistent with other variables before using plume equation.
## Table of atmospheric stability classes

<table>
<thead>
<tr>
<th>Surface wind speed</th>
<th>Day</th>
<th>Night</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Incoming solar radiation</td>
<td></td>
</tr>
<tr>
<td>mph</td>
<td>m/s</td>
<td>strong</td>
</tr>
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<td>------</td>
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<td>--------</td>
</tr>
<tr>
<td>&lt; 4.5</td>
<td>&lt; 2</td>
<td>A</td>
</tr>
<tr>
<td>4.5 – 6.7</td>
<td>2 – 3</td>
<td>A-B</td>
</tr>
<tr>
<td>6.7 – 11.2</td>
<td>3 – 5</td>
<td>B</td>
</tr>
<tr>
<td>11.2 – 13.4.7</td>
<td>5 – 6</td>
<td>C</td>
</tr>
<tr>
<td>&gt; 13.4</td>
<td>&gt; 6</td>
<td>C</td>
</tr>
</tbody>
</table>