This assignment will be due on Wednesday, March 13 at the beginning of class. Remember to show your reasoning and name the classmates you worked with. Answers without work shown will receive almost minimal credit.

(1) A UConn student claims that she can tell Dairy Bar ice cream from Herrell’s ice cream. As a test, she is given ten samples of ice cream (each sample is either from the Dairy Bar or Herrell’s) and asked to identify each one. She is right eight times. What is the probability that she would be right exactly eight times if she guessed randomly for each sample?

Solution. This should be modeled using a binomial random variable $X$, since there is a sequence of trials with the same probability of success in each one. If she guesses randomly for each sample, the probability that she will be right each time is $\frac{1}{2}$. Therefore,

$$P(X = 8) = \binom{10}{8} \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 = \frac{45}{2^{10}}.$$

(2) 20% of all students are left-handed. A class of size 20 meets in a room with 18 right-handed desks and 5 left-handed desks. What is the probability that every student will have a suitable desk?

Solution. For each student to have the kind of desk he or she prefers, there must be no more than 18 right-handed students and no more than 5 left-handed students, so the number of left-handed students must be between 2 and 5 (inclusive). This means that we want the probability that there will be 2, 3, 4, or 5 left-handed students. We use the binomial distribution and get

$$\sum_{i=2}^{5} \binom{20}{i} \left(\frac{1}{5}\right)^i \left(\frac{4}{5}\right)^{20-i}.$$

(3) Suppose that the probability of a defect in a foot of Cascade 220 Heathers yarn is 0.003. Compute the probability that a 220-yard skein of this yarn will have no defects.

Solution. There are two reasonable ways of doing this problem. We could look at it as a binomial random variable $X$. Here, our $p$ is 0.003, so we have

$$P(X = 0) = \binom{660}{0} (.997)^{660} (.003)^0 \approx .1377.$$

On the other hand, we could model this scenario by a Poisson random variable $X$. (To do this, we would have to assume that the average number of defects per foot was 0.003.)
know that for one foot, $\lambda = .003$, so for 220 yards of yarn, our $\lambda$ will be $220(3)(.003) = 1.98$. Since we want this skein of yarn not to have any defects, we want

$$P(X = 0) = e^{-1.98} \frac{1.98^0}{0!} \approx .1381.$$  

I’ll accept either method. As I said in class, the Poisson is essentially an approximation to the binomial when $p$ is small and $n$ is large, and $p = .003$ and $n = 660$ certainly qualify.

(4) Imogen is baking cookies. She mixes 400 raisins and 600 chocolate chips into her cookie dough and ends up with 500 cookies.
(a) Find the probability that a randomly picked cookie will have three raisins in it.
   
   **Solution.** This calls for a Poisson random variable $R$. The average number of raisins per cookie is $.8$, so we take this as our $\lambda$. We are asking for $P(R = 3)$, which is $e^{-0.8} \frac{0.8^3}{3!} \approx .0383$.

(b) Find the probability that a randomly picked cookie will have at least one chocolate chip in it.

   **Solution.** This calls for a Poisson random variable $C$. The average number of chocolate chips per cookie is $1.2$, so we take this as our $\lambda$. We are asking for $P(C \geq 1)$, which is $1 - P(C = 0) = 1 - e^{-1.2} \frac{1.2^0}{0!} \approx .6988$.

(c) Find the probability that a randomly picked cookie will have no more than two bits in it (a bit is either a raisin or a chocolate chip).

   **Solution.** This calls for a Poisson random variable $B$. The average number of bits per cookie is 2, so we take this as our $\lambda$. We are asking for $P(B \leq 2)$, which is $P(B = 0) + P(B = 1) + P(B = 2) = e^{-2} \frac{2^0}{0!} + e^{-2} \frac{2^1}{1!} + e^{-2} \frac{2^2}{2!} \approx .6767$.

(5) A roulette wheel has 38 numbers on it: the numbers 0 through 36 and a 00. Suppose that Jeremiah always bets that the outcome will be a number between 1 and 16 (including 1 and 16). What is the probability that Jeremiah will first win on his sixth bet?

   **Solution.** This calls for a geometric random variable $X$: Jeremiah will perform a sequence of actions until a certain event happens, and the probability of that event is the same each time. Jeremiah’s probability of success is $\frac{16}{38}$, so the probability that his first win will happen on the sixth event is $P(X = 6) = (1 - \frac{16}{38})^5 \frac{16}{38}$.

   **Suggested problems:** Chapter 4 Problems: 33, 35, 39-42, 49, 51-55, 72; Chapter 4 Theoretical Exercises: 20, 27