This assignment will be due on Thursday, September 6 at the beginning of class. Remember to show your reasoning and name the classmates you worked with. Your solutions should be detailed enough that any of your teammates could understand them simply by reading them.

(1) (Chapter 1, #42 and #44) Negate each expression and simplify your answer.
   (a) \( \forall x((P(x) \land Q(x)) \rightarrow R(x)) \)
   (b) \( \exists y(P(x) \land Q(y)) \)

(2) (Chapter 1, #74) If \( S \) and \( T \) are sets, the statement \( S = T \) can be expressed as

\[ \forall x(x \in S \leftrightarrow x \in T). \]

What does \( S \neq T \) mean? How would you go about showing that two sets are not the same?

(3) (Additional problem #0) Write your answers to each part of this question in ordinary language (for instance, writing “It is not the case that . . .” is not acceptable as a way of negating a sentence). Read your answers out loud to yourself to see if they sound natural!
   (a) Negate the sentence “All roses are red and all violets are blue.”
   (b) Write the contrapositive of “If the absolute value function is continuous at a point, then it is differentiable at that point.”
   (c) Write the converse of “If \( n \) is prime, then \( n = 2 \) or \( n \) is odd.”

(4) (Additional problem #1) A set of statements is said to be consistent if there is an assignment of truth values (that is, the same row in all their truth tables) that makes all of them true. Is the set of statements consisting of \( P \rightarrow (Q \lor R) \), \( \sim P \land Q \), and \( P \leftrightarrow R \) consistent?

(5) (Additional problem #2) For each of the following statements, say whether it is true or false and give a short explanation (one sentence is fine!).
   (a) \( \emptyset \in \emptyset \)
   (b) \( \emptyset \subseteq \emptyset \)
   (c) \( \emptyset \in \{\emptyset\} \)

(6) (Additional problem #3)
   (a) Show that the following sentences are equivalent (that is, that they have the same truth tables): \( P \rightarrow Q \) and \( \sim (P \land \sim Q) \).
(b) Now you know that proving either of the above is equivalent to proving the other. Which proof method from Section 1.5 would you be using if you proved \( \sim(P \land \sim Q) \) instead of \( P \rightarrow Q \)? Explain your answer.

Suggested problems: Chapter 1: 11-12, 17-21, 22-23, 29-32, 33-38, 45-48, 53-54, 73, 75, 82-84

Here are a few pointers for writing proofs.

- There’s a rule for giving presentations: “Tell them what you’re going to tell them, tell them what you tell them, and tell them what you told them.” The same holds for writing proofs! Tell me at the beginning what you’re going to prove. Then, after you write down the proof, explain that you’ve reached the correct conclusion. The last part may sound silly, but it’s a way of letting me know that you know you’ve done everything you needed to do.

- I should never have to wonder what kinds of things you’re talking about. If you use a variable \( x \), for instance, I need to know what kind of object \( x \) is. Is it a natural number? A real number? A function? If it is a function, what is its domain and what is its codomain?

- You should show all the steps in your reasoning, but it’s OK to condense steps in your calculations. For instance, if you want to simplify the formula

\[
2ax + 2by - 2ac + 5b + 4ac - 5b,
\]

you can just write

\[
2(ax + by + ac)
\]

without showing the intermediate algebraic steps.

- A good rule of thumb is this: imagine your proof is being shown to one of your classmates. You aren’t allowed to explain anything orally—your proof is being evaluated solely based on what you’ve written down. Are you sure your classmate can follow your proof? Might you be asked how you got from one point to another? If the answer to this last question is yes, perhaps you should explain your reasoning more carefully.

- Presentation matters. Please make your homework easy for me to read—don’t crunch your work into too small a space! I will make a donation to the Arbor Day Foundation at the end of the term in your honor if you do this.