(1) Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is chosen. If the outcome is tails, then a ball from urn B is chosen. Suppose a white ball is selected, what is the probability that the coin landed tails?

Answer: \( \frac{3}{15} + \frac{5}{12} \)

Reasons: Let \( T \) be the event that the coin landed tails, and \( W \) be the event that a white ball is selected. We want to find \( P(T|W) \).

Now, \( P(T|W) = \frac{P(TW)}{P(W)} = \frac{P(W|T)P(T)}{P(W|T)P(T) + P(W|T^c)P(T^c)} \)

\[ = \frac{\frac{3}{15} \frac{1}{2}}{\frac{3}{15} \frac{1}{2} + \frac{5}{12} \frac{1}{2}} \]

\[ = \frac{3}{15} + \frac{5}{12} \]

(2) Each of 2 balls is painted either black or red and then placed in an urn. Suppose that each ball is colored black with probability 1/3 and colored red with probability 2/3.

(a) Find the probability that there is at least 1 black ball.

(b) Suppose we know that at least 1 ball is black, what is the probability that the other ball is also black.

Answer: (a) 5/9 (b) 1/5

Reasons: (a) The probability that both balls are red is \((\frac{2}{3})^2 = 4/9\). Hence the probability that there is at least 1 black ball is \(1 - 4/9 = 5/9\).

(b) Let \( E \) be the event that there is at least 1 black ball, and \( F \) be the event that both balls are black. We want to compute \( P(F|E) \).

Now, \[ P(F|E) = \frac{P(FE)}{P(E)} = \frac{P(F)}{P(E)} = \frac{\frac{1}{3} \frac{1}{3}}{\frac{5}{9}} = 1/5 \]