Interest Rate Modeling With Random Regimes

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INTERWOVEN THEMES AND HISTORY

INTEREST RATES

STRESS TESTING

MODELS
20 Year U.S. Treasury Rates
1974: Guaranteed Income Contracts in the Group Pension Market
ANECDOTAL HISTORY

Early 1978: 12% Will Bring Blood In the Streets

20 year risk-free rate

12% will bring blood in the streets
February 1980: It Didn’t; October 1981: Even 15.13% Didn’t
ANECDOTAL HISTORY

FAILURE TO "SEE" A PLAUSIBILITY WAS CATASTROPHIC FOR INSURERS

- One Of The Largest Was Bankrupted
  - An Acquisition Disguised The Fact

- At Least One Of The Largest Visited The Fed
  - ... For A "Maybe, What If?" Discussion
  - Turned Out To Be Unnecessary
Saving Grace: Double-Digit Rates Only Lasted For 6 Years

20 year risk-free rate

OCTOBER 1979

NOVEMBER 1985
SET TODAY’S STAGE

- Dynamic Valuation Interest Rates
- Asset Adequacy Analysis
- Appointed Valuation Actuary
- Principles-Based Discussion Began
January 1993: The Fed Won’t Tolerate Long Rates Below 4%
January 1999: Maybe They Will, But That’s What RBC Is For
January 2003: No, That’s What Asset Adequacy Reserves Are For
The Concept of Stress-Testing

- STRESS-TESTING CAN APPLY TO
  - Reserves
    - Asset Adequacy Testing
    - Risk Management for Product/Line of Business
  - Surplus
    - Risk Based Capital
    - Embedded Value
  - Economic Capital
    - Basel II (III, etc.)
    - Own Solvency and Risk
  - Enterprise Risk Management
The Concept of Stress-Testing

STRESS-TESTING IS NOT

- A BY-PRODUCT OF FORECASTING
  
  - Forecasting Looks For Most Likely Outcomes
    - Maybe Within A Confidence Band
  
  - Forecasting Supports Current Decision-Making
  
  - Forecasting Will Be Judged By Actual Accuracy
The Concept of Stress-Testing

STRESS-TESTING IS NOT

- A BY-PRODUCT OF PRICING

  - Pricing Looks For Expected Values
    - Usually With Reasonable Variance Bounds
  
  - Pricing Supports Product Portfolio Development
  
  - Pricing Will Be Judged By Average Accuracy
The Concept of Stress-Testing

STRESS-TESTING IS

- A SEPARATE, DISTINCT DISCIPLINE
  - One That Looks For Extreme Values
    - Beyond Reasonable Variance or Confidence
    - But Within The Realm Of Plausibility ( ??? )
    - ...(any fool can assume that the sky will fall)
  - One That Supports Institutional Resilience
  - One That Will Be Judged By "No Surprises"
The Concept of Stress-Testing

STRESS-TESTING REQUIRES

- Truly Severe Values
  - Threats To Survival
    - Firm Not Providing Value If These Are Not "In Sight"?
    - ...(Or Maybe We’re Not "Seeing" Very Well?)
  - On Both Extremes
- That Are Somehow Still Plausible
  - By What Standard?
    - History?: at a minimum
    - Theory?: maybe
    - Judgment?: be very wary of setting a maximum
  - Informed By History, Theory, and Judgment
The Concept of Stress-Testing

STRESS-TESTING CAN / SHOULD IGNORE

- Accuracy
  - Around Likely Or Expected Scenarios

- Current Wisdom & Judgment
  - About Variance And Confidence Bands

- The Arbitrage-Free Shibboleth
  - If Someone Couldn’t Get Rich Is It Truly Extreme?
  - But Do Preserve Both Extremes

- Risk-Neutral Modeling
  - Risk-Neutral Models Predict Today’s Prices
  - Risk-Neutral Distributions Are Make-Believe
Deterministic Stress-Testing

Deterministic Interest Scenarios

- Necessary But Maybe Not Sufficient
- Risk That It’s Limited By Current Imagination
  - "There Would Be Blood In The Streets"
- Risk That It’s Limited By Historical Extremes
  - But If It Already Happened Isn’t Worse Plausible?
- How Do We Know How Bad Is Bad Enough?
- Yet Still Plausible
12% Exceeded The Bounds Of Both History And Imagination

1978: 12% will bring blood in the streets
Imagination and History Were Not Nearly Enough
Deterministic Stress-Testing

Deterministic Interest Scenarios

- Necessary But Maybe Not Sufficient
- Risk They Are Limited By Current Imagination
  - "There Would Be Blood In The Streets"
- Risk They Are Limited By Historical Extremes
  - But If It Already Happened Isn’t Worse Plausible?
- How Do We Know How Bad Is Bad Enough?
  - Yet Still Plausible
  - OR HOW LONG IS LONG ENOUGH? (Plausibly)
Remember: Double-Digit Rates Only Lasted For 6 Years
Japan Told Us That < 2% or 3% Deterministic Was Plausible

20 year risk-free rate

January 1999: 5.45%
So 2008 & 12 Were Not A "Surprise" For The Deterministic Stress-Test
Back To The Anecdotes - 1990’s

BUT FOR HOW LONG < 2% or 3% ?

- Forever?: Not Plausible
  (Remember 6 Years Of Double-Digit Rates?)

- 5 - 10 Ten Years?: Maybe Not Severe Enough?

- We Finally Resorted To Random Scenarios

- Definitely A Last Resort
  - We’d Seen Too Much Abuse of Stochastic Models
  - They Only Give Back What You Put In
    - But Hard To Recognize Own Input Coming Back At You
  - No-Arbitrage and Risk-Neutral All The Rage
    - Risked Confusing Even Knowledgeable Audience
RANDOM INTEREST RATE SCENARIOS

- The Extreme Scenarios Will Be The Stress-Test
  - So Risk-Neutral And Arbitrage-Free Are Irrelevant

- Start With A Model For An Anchor Rate
  - 20 Year Treasury
  - Build A Yield Curve Off That Later

- Choices
  - Pure Dispersion (Random Walk)
PURE DISPERSION – RANDOM WALK — IMPLAUSIBLE

\[ \Delta \ln Rate_t = Gaussian\Delta \]
RANDOM INTEREST RATE SCENARIOS

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- Start With A Model For An Anchor Rate
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- Choices
  - Pure Dispersion (Random Walk) ... Implausible
  - Introduce A Mean Reversion Point (MRP)
INTRODUCE A MEAN REVERSION POINT (MRP)

$$\Delta \ln Rate_t = F \times (MRP - \ln Rate_{t-1}) + (1 - F) \times Gaussian\Delta$$
Random Stress-Testing

RANDOM INTEREST RATE SCENARIOS

- The Extreme Scenarios Will Be The Stress-Test
  - So Risk-Neutral And Arbitrage-Free Are Irrelevant

- Start With A Model For An Anchor Rate
  - 20 Year Treasury
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- Choices
  - Pure Dispersion (Random Walk) ... Implausible
  - Introduce A Mean Reversion Point (MRP)
    - Which One (MRP) and How Fast (F)?
    - Any Choices Eliminate Some Historical Extremes –
      (Either Level Extremes &/Or "How Long?" Extremes)
    - AAA Generator Chose This (And Eliminated Both)
RANDOM INTEREST RATE SCENARIOS

- More Choices
  - Introduce More Than One MRP (Regimes)
    - Switch Off Among Them (Somehow Randomly)
INTRODUCE MORE THAN ONE MRP (REGIMES)

\[ \Delta \ln Rate_t = F \times (MRP_i - \ln Rate_{t-1}) + (1 - F) \times Gaussian \Delta \]

\( i = 1,2 \) deterministic when regime switch randomly occurs
RANDOM INTEREST RATE SCENARIOS

- More Choices
  - Introduce More Than One MRP (Regimes)
    - Switch Off Among Them (Somehow Randomly)
    - How Many? At What Levels? With What Frequency?
    - Assumptions & Output Both Look Artificial
    - Little Or No Guidance From Interest Rate History
    - ... (How much worse than 15% / 2% is plausible?)
RANDOM INTEREST RATE SCENARIOS

More Choices

- Introduce More Than One MRP (Regimes)
  - Switch Off Among Them (Somehow Randomly)
  - How Many? At What Levels? With What Frequency?
  - Assumptions & Output Both Look Artificial
  - Little Or No Guidance From Interest Rate History
  - ... (How much worse than 15% / 2% is plausible?)

- Introduce A Random Field Of MRPs (Regimes)
  - Switch Off Among Them (Somehow Randomly)
INTRODUCE A RANDOM FIELD OF MRP\(_t\)s (REGIMES)
\[
\Delta \ln \text{Rate}_t = F \times (\text{MRP}_t - \ln \text{Rate}_{t-1}) + (1 - F) \times \text{Gaussian\Delta}
\]
\(\text{MRP}_t\) random when regime switch randomly occurs
Random Stress-Testing

RANDOM INTEREST RATE SCENARIOS

- More Choices
  - Introduce More Than One MRP (Regimes)
    - Switch Off Among Them (Somehow Randomly)
    - How Many? At What Levels? With What Frequency?
    - Assumptions & Output Both Look Artificial
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    - ... (How much worse than 15% / 2% is plausible?)
  - Introduce A Random Field Of MRPs (Regimes)
    - Switch Off Among Them (Somehow Randomly)
    - Output Starts To Look Very Natural / Assumptions(??)
    - Historical Extremes Fit Right In
    - We Chose This One – Parameters A Challenge
History of 20 Year US Treasury Rate

Plausible By Definition
Neither Early 80’s Nor Japan Are Remotely Plausible In AAA
Update To New MRP Makes It Worse Even With Higher Starting Rate

20 yr risk free rate history April 1953 to February 2014 vs AAA starting January 2014
No One Scenario Hugs The Bottom – Here’s 99% Cumulative < 4% Run
AAA’s Recommended MRP Helps A Little, But Loses At The Bottom
Extreme Enough To Envelop History – But Still Plausible
M’ly %-iles Randomized MRPs / Dec. 2013 Start

Starts Higher But Still Has Similar Range Of Plausibilities

Proposed New Model Dec. 2013 - 60 Years

20 year treasury rate
And Model Design Has Not Automatically Ruled Out Bottom-Hugging

99th Percentile Worst Scenario: No. 866 - Random MRP Opens Up Long Runs < 3%
The High-Rate Risk Is Captured Much Better
FROM 1994 TO 2006:
— A LOT OF TRIAL & ERROR

SINCE THEN:
— SOME ATTEMPT AT SCIENCE
Historical Regimes – Derived From A Filtering Procedure
WAITING TIME TO REGIME SWITCH

- The MLE Gamma Distribution On Historical
  - Only 8 Data Points
    - Alpha = 3.52;  Beta = 2.32
    - Mean = Alpha times Beta = 8.2 Years
  - Alpha And Beta Low Confidence Separately
    - Mean Is Really What Affects Model Output Anyway
  - Interesting That Mean = US Political Cycle
DISTRIBUTION OF MRP
- Assume Lognormal - Mutually Independent

REVERSION SPEED
- Set Jointly With The Lognormal Parameters
- To Get Best Fit With Moments Of Historical Rates
Rate Levels and Spread Align With History

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<tr>
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<th>61 Year History</th>
<th>Model Mean</th>
<th>Model StdDev</th>
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<tr>
<td>Rate = 20 Year Treasury</td>
<td>Rate Mean</td>
<td>.0631</td>
<td>.0638</td>
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<td>Rate StdDev</td>
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<td>Rate Kurtosis (normal=3)</td>
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<td>2.92</td>
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<td>Rate 6th-osis (normal=15)</td>
<td>21.7</td>
<td>15.5</td>
<td>19.3</td>
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<tr>
<td>(6th Ctrl Mom/StdDev^6)</td>
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VOLATILITY OF INTEREST RATES

- History Is Not Clearly Lognormal
  - We Fit A Three Parameter DiGeneralized Gamma
  - Using L1 & L2 Distances Of Cumulative Distributions

- Essentially As Good As AAA Generator
  - Which Fits Historical Volatility Very Well Indeed
  - Using Stochastic Volatility & Yield Curve Dynamics
With Thanks To

- Many Generations of Actuarial Students
  - At Aetna Life Insurance Company
  - At AnTai Life Insurance Company (in Taiwan)
  - At Aetna International Inc.

- Many Generations of UConn Students
  - Master’s In Mathematics, conc. Actuarial Science
  - Master’s In Applied Financial Mathematics

- Most Recently
  - Songchen (Darren) Zhang
  - Zepeng (Ben) Xie
  - Xuezhi (Kevin) Zhang
  - Nyan Paing Tin