Improper Integrals

Solutions should show all of your work, not just a single final answer.

1. For $a > 0$, compute the improper integrals $\int_{0}^{\infty} e^{-ax} \, dx$ and $\int_{0}^{\infty} xe^{-ax} \, dx$. Express your answers in terms of $a$. (These computations are important in probability theory.)

2. Compute $\int_{5}^{\infty} \frac{dx}{x^2 - 4x + 3}$ using partial fractions. (You found an antiderivative of $1/(x^2 - 4x + 3)$ on an earlier worksheet.)

3. Decide if $\int_{0}^{\infty} \frac{x}{x^2 + 1} \, dx$ is convergent or divergent. If it is convergent, evaluate it.

4. Determine if $\int_{1}^{\infty} \frac{\cos x}{x^2} \, dx$ is convergent or divergent.

5. For which values of $p > 0$ is the integral $\int_{2}^{\infty} \frac{dx}{x(\ln x)^p}$ convergent?

6. T/F (with justification) The integral $\int_{0}^{2} \frac{dx}{x - 1} \, dx$ is convergent.