Integration by Substitution (Review)

**Part 1:** Compute the indefinite integral in three steps: (1) specify the substitution $u = ?$ and $du = ?$, (2) rewrite the integral completely in terms of the new variable $u$, (3) compute the new integral and express the final answer in terms of the original variable.

**Example.** Evaluate $\int x \frac{1}{x^2 + 1} dx$.

**Solution.**

1. Set $u = x^2 + 1$, so $du = 2x \, dx$.
2. Since $x \, dx = \frac{1}{2} du$, we have $\int \frac{x}{x^2 + 1} \, dx = \int \frac{(1/2) du}{u} = \int \frac{1}{2} \cdot \frac{1}{u} \, du$.
3. $\int \frac{1}{2} \cdot \frac{1}{u} \, du = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln |x^2 + 1| + C = \frac{1}{2} \ln(x^2 + 1) + C$.

1. Evaluate $\int x^2 \sin(x^3) \, dx$.

2. Evaluate $\int x \sqrt{4x + 1} \, dx$. 
3. Evaluate $\int \frac{1 + x}{1 - x} \, dx$.

4. Evaluate $\int \frac{1}{x \ln x} \, dx$.

5. Evaluate $\int \sin^3 x \, dx$. (Hint: $\sin^2 x = 1 - \cos^2 x$.)
Part 2: Express the definite integral in $x$ as a new definite integral in the new variable $u$. Don’t evaluate the new definite integral.

**Example.** Rewrite $\int_2^3 xe^{-x^2} \, dx$ in terms of $u = x^2$.

**Solution.** When $u = x^2$, $du = 2x \, dx$. If $x = 2$ then $u = 4$, and if $x = 3$ then $u = 9$, so

$$\int_2^3 xe^{-x^2} \, dx = \int_{u=4}^{u=9} e^{-u} \frac{du}{2} = \frac{1}{2} \int_4^9 e^{-u} \, du.$$

6. Rewrite $\int_0^1 (3x + 1)^2 \, dx$ in terms of $u = 3x + 1$.

7. Rewrite $\int_0^1 x^2(1 + 2x^3)^5 \, dx$ in terms of $u = 1 + 2x^3$.

8. Rewrite $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} \, dx$ in terms of $u = \cos x$.

9. Rewrite $\int_0^{\pi/3} \sin x \cos x \, dx$ in terms of $u = \cos x$.

10. Rewrite $\int_0^{\pi/3} \sin x \cos x \, dx$ in terms of $u = \sin x$. 