Algebra

Name: ___________________________  Section No: _________

In your solutions to the exercises below, show the algebraic work that leads to the final answer.

Simplifying Algebraic Expressions

1. Simplify the expression $\frac{1}{3-2} - \frac{1}{3} + \frac{1}{4-1}$.

2. Simplify the expression $\frac{(x^2y^{-3})^2}{(y^{-3}x^{-2})^{-2}}$.

3. Simplify $(4x^6)^{3/2}$.

4. If $f(x) = x^2 + 3x$ and $h \neq 0$, then simplify $\frac{f(x + h) - f(x)}{h}$.

5. Simplify $\frac{1}{x + h} - \frac{1}{x}$.

6. Rationalize $\frac{3}{x - \sqrt{x}}$.

Intervals

7. Write in interval notation
   
   (a) the open interval with endpoints 2 and 3,
   (b) the closed interval with endpoints 2 and 3,
   (c) the half-open interval with endpoints 2 and 3 that contains 2 but not 3.

8. Represent the following sets of numbers in two ways: interval notation and drawn as subsets of the number line:
   
   (a) $1 \leq x < 3$,  (b) $-2 \leq x \leq 2$,  (c) $-5 < x \leq -3$. 
Completing the Square

9. Complete the square for the following expressions:
   (a) \(x^2 - 8x + 12\), \hspace{1em} (b) \(s^2 + 3s - 6\).

10. Write the following expressions as a difference of squares:
    (a) \(x^2 + 4x\), \hspace{1em} (b) \(y^2 + 5y\).

Solving Equations

11. Solve for \(x\) in terms of \(y\): \(2y^2x - y^2 - (1 + 3y) = x\).

12. Find the solutions of \(\frac{x^2}{3} + 2x - 1 = 0\) exactly (that is, not approximations).

13. Find the solutions of \(\frac{1}{x - 4} + \frac{1}{x + 4} = \frac{4}{x^2 - 16}\) exactly.

14. Find the solutions of \(\frac{1}{x} - \frac{1}{x + 2} = 2\) exactly.

Factoring

15. Factor \(x^2 - 2x - 24\).

16. Factor as much as possible \(x^3 - a^2x\).

Exponential and Logarithmic Functions

17. Simplify
   (a) \(\frac{2^{5x}}{2^x}\) \hspace{1em} (b) \(e^{2x}e^{-3x}\) \hspace{1em} (c) \(\frac{e^{2x} - 1}{e^x - 1}\) \hspace{1em} (d) \(\sqrt[3]{5^{2x}}\).

18. Evaluate \(\log_4(1/64)\).

19. Solve for \(x\) in the equation \(\log_2 x + \log_2(x - 2) = 3\).

20. Solve for \(t\) in the equation \(\ln t - \ln(t^2) = 5\) exactly.

Trigonometric Functions

21. On the unit circle mark off the following angles (in radians):
    (a) \(\frac{\pi}{2}, \pi, \text{ and } -\frac{\pi}{2}\) together \hspace{1em} (b) \(\frac{\pi}{3}\) and \(\frac{2\pi}{3}\) together.
22. Evaluate the following, where \( k \) is an integer in part d.

(a) \( \sin \left( \frac{7\pi}{2} \right) \) \hspace{1cm} (b) \( \cos \left( \frac{-\pi}{2} \right) \) \hspace{1cm} (c) \( \sin (101\pi) \) \hspace{1cm} (d) \( \sin \left( \frac{\pi}{2} + 2k\pi \right) \).

**Inverse Functions**

23. Find the inverse of each of the following functions, including the domain, if an inverse exists.

(a) \( f(x) = \frac{x}{1+2x} \) for \( x \neq -\frac{1}{2} \) \hspace{1cm} (b) \( f(x) = \sqrt{18-2x^2} \) for \( 0 \leq x \leq 3 \).

24. Find the inverse of each of the following functions, including the domain, if an inverse exists.

(a) \( f(x) = \ln(e^{2x} + 1) \) for all \( x \) \hspace{1cm} (b) \( f(x) = \frac{e^x}{1+2e^x} \) for all \( x \).

**Graphs**

25. Below is a graph of \( y = f(x) \). Sketch the following graphs, using a new set of axes each time. Pay attention to translation, compression, and stretching of the graph.

(a) \( y = f(x + 1) \) \hspace{1cm} (b) \( y = f(x - 1) \) \hspace{1cm} (c) \( y = f(2x) \) \hspace{1cm} (d) \( y = 2f(x) \).

![Graph of y = f(x)](image.png)