This sample exam is just a guide to prepare for the actual exam. Questions on the actual exam may or may not be of the same type, nature, or even points. Don’t prepare only by taking this sample exam. You also need to review your class notes, homework and quizzes on WebAssign, quizzes in discussion section, and worksheets.

The exam will cover the following sections (from the course outline): 3.4, 3.5, 3.9, 3.10, 1.5, 3.6, 3.8, 4.8, 4.1, 4.2, 4.3, 4.4.

Read This First!

- Please read each question carefully. Other than the question of true/false items, show all work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.

- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.

- Unless instructed otherwise, give any numerical answers in exact form, not as approximations. For example, one-third is $\frac{1}{3}$, not .33 or .33333. And one-half of $\pi$ is $\frac{1}{2}\pi$, not 1.57 or 1.57079.

<table>
<thead>
<tr>
<th>Grading - For Administrative Use Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question: 1</td>
</tr>
<tr>
<td>Points:</td>
</tr>
<tr>
<td>Score:</td>
</tr>
</tbody>
</table>
1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

(a) A local maximum value of a function is also an absolute maximum value. T F [3]

Justification:

(b) If \( f(x) \) is differentiable then the derivative of \( f(e^x) \) with respect to \( x \) is \( f'(e^x)e^x \). T F [3]

Justification:

(c) Every continuous function on \((0, 1)\) has an absolute maximum value. T F [3]

Justification:

(d) If \( \frac{dy}{dx} = y \), then \( y = 0 \) or \( y = e^x \). T F [3]

Justification:

(e) The graph of \( y = \ln(x^2) \) for \( x > 0 \) is concave down. T F [3]

Justification:
2. (a) Compute \( \frac{d}{dx} \left( \frac{\ln^2(x)}{e^x - 1} \right) \). You do not need to simplify. \[3\]

(b) Find the derivative of \( y = 5^x \) in two ways and check they agree: \[3\]

(i) express \( 5^x \) as a power of \( e \) and use the fact that \( (e^x)' = e^x \).

(ii) logarithmic differentiation, writing the final answer entirely in terms of \( x \).
3. Use implicit differentiation to find the equation of the tangent line to the graph of \( y^2 = x^3 + 2xy \) at the point \((3, -3)\), as marked below. Write the final answer in the form \( y = mx + b \).
4. (a) Find the linearization of $x\sqrt{x}$ at 4.

(b) Use the linear approximation obtained in part a, not other methods, to approximate $5\sqrt{5}$.
(You must use part a. A value of $5\sqrt{5}$ purely from a calculator will earn 0 points.)
5. (a) When a particle with mass $m$ has velocity $v$, its kinetic energy is $K = \frac{1}{2}mv^2$. At what rate is the kinetic energy changing for a particle of mass 5 kg when its velocity is 10 m/sec and its acceleration is 3 m/sec$^2$? (Note: mass is constant: it does not change over time.)

(b) Two boats leave a dock at the same time. One boat travels north at 30 mi/hr and the other travels east at 40 mi/hr. After half an hour how quickly is the distance between the boats increasing, in mi/hr?
6. A pile of the radioactive substance Unobtainium loses 6\% of its mass in a year.

(a) Let $U(t)$ denote the amount of Unobtainium in kilograms that remains in a sample after $t$ years. If the initial mass is 50 kg, determine the differential equation for the rate of change of $U(t)$ and also a formula for $U(t)$.

(b) Find the half-life of Unobtainium in years, accurate to 3 decimal places.
7. Use calculus to find the absolute maximum value of the following functions on the indicated intervals. Justify your final answers, which must be given exactly.
(a) \( f(x) = \sin x + \cos x \) on \([0, \pi]\) 
(b) \( f(x) = (7x - 1)e^{-2x} \) on \([0, 1]\)
8. When $f(x) = \frac{x^2}{x^2 - 1}$, use calculus to find 

(i) the critical numbers of $f(x)$,
(ii) the open intervals where $f(x)$ is increasing and where $f(x)$ is decreasing,
(iii) the open intervals where the graph of $y = f(x)$ is concave up and concave down.

**Note:** Remember to pay attention to points where $f(x)$ is not defined.
9. (a) Find the linearization of the function \( f(x) = x^4 + 4x^2 \) at 1. \[4\]

(b) Find the linearization of the function \( g(x) = \sqrt[5]{1 + x} \) at 0. \[4\]

10. (a) Use Newton’s method to find the fourth approximation to the root of \( x^3 + 3x - 3 = 0 \) when \( x_1 = 2 \). Round your final answer to 3 decimal places. \[3\]

(b) Using Newton’s method to solve \( x^5 - x - 7 = 0 \) when \( x_1 = 1 \), find the least \( n \) such that \( x_n \) and \( x_{n+1} \) agree to three decimal places. \[3\]
11. Here are three theorems about continuous functions. Draw a picture that illustrates each theorem, using the notation of the theorem in your picture.

(a) **Extreme Value Theorem**: If $f(x)$ is a continuous function on $[a, b]$ then it has an absolute maximum value and an absolute minimum value on $[a, b]$.

(b) **Rolle’s Theorem**: If $f(x)$ is a continuous function on $[a, b]$ that is differentiable on $(a, b)$, and $f(a) = f(b)$, there is a $c \in (a, b)$ such that $f'(c) = 0$.

(c) **Mean Value Theorem**: If $f(x)$ is a continuous function on $[a, b]$ that is differentiable on $(a, b)$, there is a $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Draw a picture for the case when $f(a) \neq f(b)$.
12. Evaluate the following limits exactly, using l’Hospital’s Rule.

(a) \( \lim_{x \to 0} \frac{5^x - 4^x}{3^x - 2^x} \) [3]

(b) \( \lim_{x \to 0} \frac{\sin^2(ax)}{x^2} \), where \( a \neq 0 \). (The answer will depend on \( a \).) [3]

(c) \( \lim_{x \to \infty} \left(1 + \frac{10}{x^2}\right)^{x^2} \) [3]