Worksheet 6: Integration

Name: ____________________________  Section No: ______________

(1) Find the most general antiderivative of the function (use $C$ as any constant).

(a) $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$

(b) $f(t) = \frac{t^4 + 3\sqrt{t}}{t^2}$

(c) $g(\theta) = \cos \theta - 5\sin \theta$

(2) Find $f(x)$ satisfying the given conditions.

(a) $f'''(x) = \cos x$, $f(0) = 1$, $f'(0) = 2$, and $f''(0) = 3$

(b) $f''(x) = 2 - 12x$, $f(0) = 9$, $f(2) = 7$

(3) The graph of $y = 4 - x^2$ over the interval $[0, 2]$ is given below.

(a) Estimate the area under the graph over $[0, 2]$ using 4 rectangles and right endpoints: sketch the rectangles and then compute their areas.

(b) Repeat part (a) using 4 rectangles and left endpoints.

(c) Repeat part (a) using 4 rectangles and midpoints.

(d) Compute the exact area under $y = 4 - x^2$ over $[0, 2]$ using the Fundamental Theorem of Calculus and indicate which of the approximations in (a), (b), and (c) is closest to this.
(4) Here’s a graph of the velocity (in ft/sec) of an object on a horizontal line.

(a) Describe the motion of the object over the interval $0 \leq t \leq 24$: when is it moving left or right, and when is it speeding up or slowing down?

(b) Compute the net change in position of the object between $t = 0$ and $t = 8$ seconds, in feet. (A positive answer means a net change to the right, a negative answer means a net change to the left, and the answer 0 means the object ends up back where it began.)

(c) Compute the net change in position of the object between $t = 8$ and $t = 20$ seconds, in feet.

(5) (a) Let $A_0(x) = \int_0^x (1 - t^2) \, dt$, $A_1(x) = \int_1^x (1 - t^2) \, dt$, and $A_2(x) = \int_2^x (1 - t^2) \, dt$. Compute these explicitly in terms of $x$ using Part 2 of the Fundamental Theorem of Calculus.

(b) Over the interval $[0, 2]$, use your answers in part (a) to sketch the graphs of $y = A_0(x)$, $y = A_1(x)$, and $y = A_2(x)$ on the same set of axes.

(c) How are the three graphs in part (a) related to each other? In particular, what does Part 1 of the Fundamental Theorem of Calculus tell you about the graphs in part (a)?

(d) How are the intervals where the graphs in part (a) are increasing and decreasing, their concavity, and their critical numbers, related to the graph of $1 - x^2$ on the interval $[0, 2]$?

(e) On a graph of $y = 1 - t^2$, for $0 \leq t \leq 2$, shade the region with signed area $A_0(1.5)$. Indicate with + and − which area counts positively and which negatively.

(6) Use a definite integral to write down a function $g(x)$ such that $g'(x) = \cos^3 x$ and $g(0) = 1$. Explain why your answer fits the required conditions.