An airliner passes over an airport at noon travelling 500 mi/hr due west. At 1:00 PM, another airliner passes over the same airport at the same elevation traveling due north at 550 mi/hr. Assuming both airliners maintain their (equal) elevations, how fast is the distance between them changing at 2:30 PM?

The trajectories of the two planes will form a right angle at the airport. Let \( x \) be the distance the westbound plane has traveled past the airport, \( y \) the distance the northbound plane has traveled past the airport, and \( z \) the distance between the two planes, as shown in the figure above. We are given that \( \frac{dx}{dt} = 500 \text{ mi/hr} \) and \( \frac{dy}{dt} = 550 \text{ mi/hr} \). Our goal is to find \( \frac{dz}{dt} \) at 2:30 PM.

We can apply the Pythagorean Theorem to get the relationship
\[
x^2 + y^2 = z^2
\]
And take the derivative with respect to time to get
\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}
\]
Solve for \( \frac{dz}{dt} \):
\[
\frac{2x \frac{dx}{dt} + 2y \frac{dy}{dt}}{2z} = \frac{dz}{dt}
\]
At 2:30 PM: the westbound plane has been flying past the airport for 2.5 hours (from 12:00 PM to 2:30 PM), so \( x = (2.5 \text{ hr})(500 \text{ mi/hr}) = 1250 \) miles. The northbound plane has been past the airport for 1.5 hours, so \( y = (1.5 \text{ hr})(550 \text{ mi/hr}) = 825 \) miles. Hence \( z = \sqrt{825^2 + 1250^2} \)

At this point we can plug in the values of \( x, y, z, \frac{dx}{dt} \) and \( \frac{dy}{dt} \).
\[
\frac{dz}{dt} = \frac{2(1250)(500) + 2(825)(550)}{2\sqrt{825^2 + 1250^2}} \approx 720.268
\]