Practice Exam 1(b) Solutions (Does not include graphing solutions)

1. (a) $y(t) = -\ln(-\frac{1}{2}t^2 + \frac{1}{e}) = \ln(\frac{-2e}{et^2-2})$

   (b) $y(t) = e^t + te^t = (t+1)e^t$

2. Let $S(t)$ denote the total amount of salt (in grams) in the tank at time t (in days). Then $S(0) = 4T/gal \times 50gal = 200T$

   (a) $\dfrac{dS}{dt} = 5\ T/gal \times 2\ gal/day - S(t)/50\ T/gal \times 2\ gal/day = 10 - \dfrac{S(t)}{25}, \quad S(0) = 200$

   (d) $S(t) \to 250$ as $t \to \infty$, $S(t) \to -\infty$ as $t \to -\infty$.

   (e) Yes. $S(t)$ would still go to 250 as $t \to \infty$. However $S(t) \to \infty$ as $t \to -\infty$ since $S(0) = 400$ in this case.

3. (a) $y(t) < -2$ for all t, $y(t)$ is increasing, $y(t) \to -2$ as $t \to \infty$, and $y(t) \to -\infty$ as $t \to -\infty$.

   (b) $y(t) = -2$ for all t.

   (c) $-2 < y(t) < 2$ for all t, $y(t)$ is decreasing, $y(t) \to -2$ as $t \to \infty$, and $y(t) \to 2$ as $t \to -\infty$.

   (d) $y(t) = 2$ for all t.

   (e) $y(t) > 2$ for all t, $y(t)$ is increasing, $y(t) \to \infty$ as $t \to \infty$, and $y(t) \to 2$ as $t \to -\infty$.

4. (a) The phase line would have equilibrium points at $y = -1$ and $y = 1$. The equilibrium at $y = 1$ would be a sink, $y = 1$ a source. (So there would be up arrows above $y = 1$, and below $y = -1$ and down arrows for y between -1 and 1.)

5. (a) From the bifurcation diagram we can see that when $\alpha > 400$ there are no equilibrium points, when $\alpha = 400$ there is a node at $y = 3000$, and when $\alpha < 400$ there are two equilibrium points. Thus a bifurcation occurs when $\alpha = 400$.

   (b) Looking at the phase line for $\alpha = 200$ we see that the solution, $P(t)$, to the IVP: $\frac{dP}{dt} = f_\alpha(P), \quad P(0) = 3000$, increases to about 5200 as $t \to \infty$. (This equilibrium point is a sink.) And so the population of squirrels settles down to about 5,200 in the long run.

   (c) If 450 squirrels are transported, (i.e. $\alpha = 450$) each year, we see that the corresponding phase line has no equilibrium points and that P is always decreasing. Hence we expect the squirrel population to die out eventually. (We're to the right of the right-most equilibrium point on the bifurcation diagram.)

6.

t_k	y_k
1.0	2.0
1.5	2.5
2.0	5.6875