Name:

Math 211 **Practice Exam**

Directions. *Answer the following questions to the best of your ability. No credit will be given for answers showing no work.*

1. **Solve the following Initial Value Problems**

   (a) $\begin{cases} \dfrac{dy}{dt} = e^y \cdot t \\ y(0) = 1 \end{cases}$

   (b) $\begin{cases} \dfrac{dy}{dt} = e^t + y \\ y(1) = 2e \end{cases}$

2. Suppose I have a 50−gallon saltwater fish tank that is leaking at a rate of two gallons per day. The saltwater fish tank is kept well mixed, so the concentration of salt is uniform throughout the tank. I am addding saltwater containing 5 T of salt per gallon at a rate of 2 gallons per day to keep the tank full. The initial concentration of salt in the tank is 4 T/gallon.

   (a) Set up an initial value problem which models this situation. **DO NOT SOLVE!**

   (b) Sketch the phase line corresponding to the ODE.

   (c) Using the above phase line, trace the approximate path of the solution which would solve your initial value problem.

   (d) What is the long-term behavior of your solution (as $t \to +\infty$ and as $t \to -\infty$)?

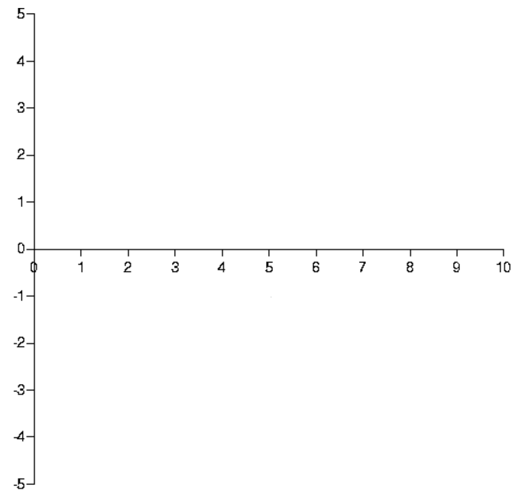
   (e) Would the long-term behavior of the solution be different if the initial concentration of salt in the tank were 8T/g? Why or why not?

3. **Existence and Uniqueness**

   Consider the autonomous equation

   $$\frac{dy}{dt} = y^2 - 4$$

   Sketch a graph of different solutions depending on initial conditions:



What does the uniqueness theorem tells us in the following cases? More precisely, is the solution a constant function, increasing or decreasing and what is the behavior for $t \to +\infty$ and $t \to -\infty$?

(a) $y(1) = -30$

(b) $y(1) = -2$

(c) $y(1) = 0$

(d) $y(1) = 2$

(e) $y(1) = 2.0000000001$

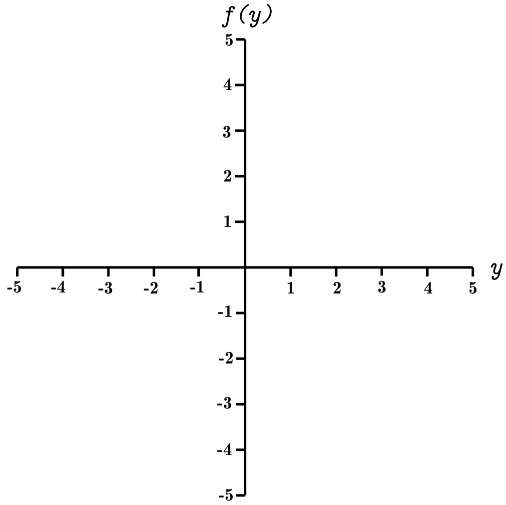
4. **The Phase Line**

Assume that we have the ODE

$$\frac{dy}{dt} = \frac{y^2 - 1}{y^2 + 1}$$

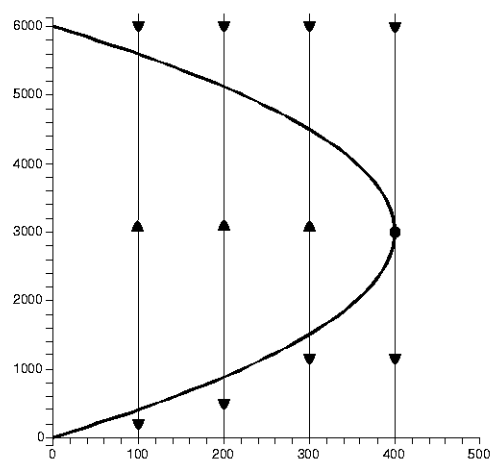
(a) What would the phase line look like?

(b) Assume that the line on the left is the phase line of the autonomous differential equation. Draw the graph of f next to it. Note: do *not* graph the solutions.



5. **The Bifurcating Squirrel Population**

A forest in Saskatchewan has an overpopulation of squirrels. To alleviate the problem, each year some squirrels are captured and transported to a forest in Pennsylvania, which is in dire need of squirrels. Let $\frac{dP}{dt} = f_\alpha(P)$ where P is the population of squirrels in Saskatchewan. Time, t, is given in years. The parameter, α, represents the number of squirrels transported to Pennsylvania every year. The graph below is a bifurcation diagram for this differential equation. Suppose there are 3000 squirrels in Saskatchewan when the transporting process begins.



(a) What is the bifurcation value for $\frac{dP}{dt} = f_\alpha(P)$?

(b) If 200 squirrels are transported each year, will the population in Saskatchewan stabilize in the long run, and if so, at what level?

(c) Same question but with 450 squirrels transported to Pennsylvania each year.

(d) Suppose the population of squirrels in Saskatchewan was reduced to 500 due to a virus. According to the diagram above, what is the largest number of squirrels that can be transported to Pennsylvania per year without causing the population of squirrels in Saskatchewan to decline to zero?

6. **Euler's Method**

Consider the initial value problem

$$\frac{dy}{dt} = y^2 t - 3, \quad y(1) = 2$$

Use Euler's Method with a step size of $\triangle t = .5$ to calculate approximate values for $y(1.5)$ and $y(2)$.