**Practice Exam 1(a) Solutions (Does not include graphing solutions)**

1. (a) $y(t) = -\cos t + 1$

   (b) $y(t) = 8e^{5t} + 3te^{5t}$

   (c) (This is from Section 1.9) $y(t) = \frac{3}{2}t^2 + \frac{1}{2}t^5$

2. (a) $\frac{dy}{dt} = y^2 + 3y + 9/4 = (y + 3/2)^2$

   Thus if $\alpha = 9/4$ we have one equilibrium point, a node at $y = -3/2$.

   If $\alpha > 9/4$ then $dy/dt > 0$ for all $y$ since $(3^2 - 4\alpha) < 0$, so there no equilibria.

   If $\alpha < 9/4$ then there are two equilibria since $(3^2 - 4\alpha) > 0$. They are at $y = \frac{-3 \pm \sqrt{3^2 - (4)(\alpha)}}{2}$.

   (c) This is partly described above. If $\alpha < 9/4$ then we have two equilibrium points located at $y$ values indicated above. In this case the lesser $y$ equilibrium point is a sink, the greater one is a souce. As $\alpha$ increases to $9/4$, the equlibria get closer and closer together, until $\alpha = 9/4$, where we have one equilibrium point, a node, at $y = -3/2$. For $\alpha > 9/4$, there are no equilibria and our solutions are always increasing since $dy/dt > 0$.

3. (b) The ferrets survive for any initial value $P_0 \geq 20$. If $P(0) = 20$ then the population is in equilibrium. If $P(0) > 20$ then $P(t) \to 60$ as $t \to \infty$ since $P = 60$ is a sink. However if $P(0) < 20$, then $P(t)$ approaches the sink at $P = 0$ as $t \to \infty$.

4.

| $t_k$ | $y_k$ |
|---|---|
| 0.0 | -0.375 |
| 0.5 | -0.5 |
| 1.0 | 0.0 |
| 1.5 | 0.5 |

5. Let $S(t)$ denote the total amount of salt (in grams) in the tank at time $t$ (in days). Then $S(0) = 4g/L \times 200L = 800g$

   (a) $\frac{dS}{dt} = \frac{Sg}{200L} \times \frac{-8L}{day} = -\frac{S}{25} \cdot \frac{grams}{day}, \qquad S(0) = 800g.$

   (d) $S(t) \to 0$ as $t \to \infty$.

   (e) No. $S(t)$ would still go to 0 as $t \to \infty$. It would just take slightly longer to do so. Note that $\frac{dS}{dt} < 0$ for any $S > 0$ so the total amount of salt approaches the equilibrium value of 0 as $t$ increases.

6. (a) (i) $y(t) > 3$ for all $t$

   (ii) $y(t) = 3$ for all $t$

   (iii) $0 < y(t) < 2$ for all $t$

   (iv) $y(t) < 0$ for all $t$

7. (a) At the point $(0, 0)$ there is a mini-tangent with a slope of 0 (horizontal).

   (b) At the point $(2, 3)$ there is a mini-tangent with a slope of 10.