Name:

Math 211 **Practice Exam 1**

Directions. *Answer the following questions to the best of your ability. No credit will be given for answers showing no work.*

1. Solve the following initial value problems:

   (a) $\begin{cases} \dfrac{dy}{dt} = \sin(t) \\ y(0) = 0 \end{cases}$

   (b) $\begin{cases} \dfrac{dy}{dt} = 5y + 3e^{5t} \\ y(0) = 8 \end{cases}$

   (c) $\begin{cases} \dfrac{dy}{dt} = \dfrac{4y}{t} - 3t \\ y(1) = 2 \end{cases}$

2. Consider the following one-parameter family of differential equations

$$\frac{dy}{dt} = y^2 + 3y + \alpha.$$

   (a) Show (algebraically) that the bifurcation value for this family of equations is $\alpha = 9/4$.

   (b) Draw three phase lines for the differential equation above - one corresponding to any α of your choice less than $9/4$, one corresponding to any α of your choice greater than $9/4$, and one corresponding to $\alpha = 9/4$. Label each equilibrium point as a sink, source, or node.

   (c) Describe how the solutions (and equilibrium points) change for this differential equation as the parameter varies.

3. Consider a population of ferrets modeled by the differential equation

$$\frac{dP}{dt} = f(P)$$

   The corresponding phase line has:

   i. a sink at $P = 0$,
   ii. a sink at $P = 60$,
   iii. and a source at $P = 20$.

   (a) Draw the phase line for this model.

   (b) What initial values of P ensure the survival of the ferrets and WHY? (only include practical values for P.)

   (c) Use the phase line to sketch the graph of $f(P)$ on the plane below and clearly label the axes.

4. Use Euler's method with the given step size $\triangle t$ to approximate the solution to the given initial-value problem over the time interval specified. Your answer should include a table of the approximate values of the dependent variable as well as a sketch of the graph of the approximate solution.

$$\frac{dy}{dt} = t^2 + y, \quad y(1) = 0, \quad 0 \le t \le 1.5, \quad \triangle t = 0.5$$

5. Tammy has a 200 l saltwater fish tank that is leaking at a rate of 8 l per day and she is adding pure water at a constant rate to keep the tank full. The initial concentration of salt in the tank is 4g/l.

   (a) Set up an initial value problem which models this situation. **do not solve**

   (b) Sketch (roughly) the phase line corresponding to the ODE.

   (c) Using the above phase line, trace the approximate path of the solution which would solve your initial value problem.

   (d) What is the long-term behavior of your solution (as $t \to \infty$)?

   (e) Would the long-term behavior of the solution be different if the initial concentration of salt in the tank were 8g/l? Why or why not?

6. Consider the following ODE:

$$\frac{dy}{dt} = y(y-2)(y-3)$$

   (a) What does the existence and uniqueness theorem say about the solution if

      i. $y(0) = 4$

      ii. $y(0) = 3$

      iii. $y(0) = 1$

      iv. $y(0) = -1$

   (b) Draw the slopefield on the (t, y) plane at $(1, 1)$, $(1, 2)$ and $(2, 1)$

   (c) Draw the phase line and sketch the graphs of solutions.

7. Let $\dfrac{dy}{dt} = yt + t^2$.

   (a) If creating the slope field for this differential equation, what would you draw at the point (0,0)?

   (b) What about at (2,3)?