MATH 276 - Actuarial Models
Spring 2008 - Valdez
Class Project 2
due Friday, 6:00 PM, 2 May 2008
Total marks: 100

Please write your names and student numbers at the spaces provided:

Student 1: ______________________________ **Student ID:** ______________________

Student 2: ______________________________ **Student ID:** ______________________

Follow these instructions:

- **There are three (3) parts here and you are to answer all parts together with the questions associated with each part.**
- **You are allowed to partner with another student in the class - indeed, you are highly recommended to work with someone else considering the expected amount of work required to complete this project.**
- **You may handwrite or type your solutions, provided you label your solutions. When asked to provide output in R (or other preferred software), please provide the command used, and label them according to the part of the questions asked. Put them as appendices. When cutting/pasting from R into a word document, use a monospace font (e.g. Courier, Courier New).**
- **If you handwrite solutions, please write legibly.**
- **You will be marked according to the accuracy and the quality of your presentation and discussion of your results.**
- **Please print and sign this front page. Your paper will not be marked unless this front page (with signatures) has been received.**
- **Unless otherwise granted with prior approval, you must submit your solutions (hardcopy) in person at room MSB 418 by the deadline.**

We certify that this is our own work, and that we have not copied the work of anybody else.

Signature 1: ______________________________ **Date:** ______________________

Signature 2: ______________________________ **Date:** ______________________

1. (40 points) Assume today is 1 January 2008. You have been hired as a consulting actuary to investigate the profitability of a small insurance company's this year's automobile line of business. The company now has 100,000 policies all issued (new or renewed) at the beginning of the year with the following characteristics.

Type of driver	Number of policies	One-year premium per policy
male, young	10,000	750
male, old	40,000	600
female, young	20,000	650
female, old	30,000	700

In addition, the portfolio has an overhead (or fixed) expense of $1,000,000 for the year, plus an additional $200,000 for your consulting fees. A per policy expense of $3.25 has been determined to cover administration and other related expenses.

After reviewing the company's historical claims experience, you have determined that the claim count , N, and the claim size, X, given claim, for each type of driver follows the following parametric distributions:

Type of driver	$N \sim \text{Poisson}(\lambda)$	$X \sim \text{Pareto}(\theta, \alpha)$
male, young	$\lambda = 2000$	$\theta = 1500, \alpha = 2$
male, old	$\lambda = 4000$	$\theta = 1500, \alpha = 2$
female, young	$\lambda = 2000$	$\theta = 1800, \alpha = 2.5$
female, old	$\lambda = 1500$	$\theta = 1800, \alpha = 2.5$

In addition, you are given the following information:

- Premiums are collected at the beginning of the year, and all expenses are incurred at the beginning of the year.
- Claims are paid at the end of the year.
- Any unused premiums to pay expenses/claims at the beginning of the year are invested at the risk-free rate of 2.5%.

Using simulation, discuss an impression of the profitability of the company's automobile insurance portfolio. Your discussion should include exploration of:

(a) the portfolio's overall aggregate claims and the aggregate claims by type of driver; and

(b) the portfolio's overall profitability and the profitability by type of driver.

Provide supporting summary statistics and graphs (e.g. histogram, density plots, etc.) in your discussion. At the minimum, give estimates of the mean of aggregate claims and mean of profit, together with a 95% confidence intervals of these estimates. Do this for each type of driver as well, to examine the balance of the premium across the different types of drivers. Be creative in your analysis and discussion.

2. (25 points) Your company is evaluating a new venture, called **ProjectBucky**. This new venture requires an initial investment of \$50 million. Your company uses the traditional discounted-cash-flow (DCF) approach to appraise capital investment projects. According to this approach, if I_0 is the amount of initial investment and CF_t are the cash flows at times $t = 1, 2, \ldots, n$, then

$$I_0 = \sum_{t=1}^{n} \frac{\mathrm{CF}_t}{(1+R)^t},$$

where R is the annual rate of return on the investment.

ProjectBucky is only a one-year investment where the projected end-of-the-year cash flow (CF) has a log-normal distribution with parameters $\mu = 3.2$ and $\sigma^2 = 1.5$ so that its mean is

$$\mathrm{E}(\mathrm{CF}) = e^{\mu+\sigma^2/2} \approx 52,$$

and variance is

$$\mathrm{Var}(\mathrm{CF}) = \left(e^{\sigma^2} - 1\right) e^{2\mu+\sigma^2} \approx 9,391.$$

The units are in millions.

Evaluate the feasibility of investing in this new venture, using simulation. In your discussion:

(a) provide a distribution graph of the return on the investment,

(b) provide useful statistics to back your discussion with at least the expected return on the investment together with a 95% confidence interval (use the bootstrap method to estimate the standard deviation of your estimated expected return), and

(c) analyze other useful statistics which encourages or discourages you to invest in the new venture.

3. (35 points) In class, we have learned about the **Gompertz** distribution to model human lifetime with a force of mortality that increases exponentially with age. A variation to this distribution is the **Makeham**'s law of mortality which has an additional constant that is independent of age. The force of mortality here is of the expression:

$$\mu_x^{\mathrm{M}} = A + Bc^x.$$

In contrast, the **Gompertz** has $\mu_x^{\mathrm{G}} = Bc^x$. Here the superscripts M and G denote Makeham's and Gompertz, respectively.

The following properties can be easily derived (you do not need to provide the proof). For an individual currently age x, its future lifetime T_x has survival distribution

$$S_{T_x}^{\mathrm{M}}(t) = \mathrm{P}(T_x > t) = e^{-At - \frac{Bc^x}{\log(c)}(c^t - 1)},$$

and density function

$$f_{T_x}^{\mathrm{M}}(t) = (A + Bc^{x+t}) e^{-At - \frac{Bc^x}{\log(c)}(c^t - 1)}.$$

For the case of Gompertz, recall from class we would have the correponding survival and density functions:

$$S_{T_x}^{\mathrm{G}}(t) = \mathrm{P}(T_x > t) = e^{-\frac{Bc^x}{\log(c)}(c^t - 1)},$$

and

$$f_{T_x}^{\mathrm{G}}(t) = Bc^{x+t} e^{-\frac{Bc^x}{\log(c)}(c^t - 1)}.$$

It is easy to simulate from the Gompertz, using the inverse transform method, again as discussed in class. Consider the case where the parameters are:

$$A = 0.001, \; B = 0.0000070848535, \text{ and } c = 1.1194379.$$

These parameter values have been suggested in Mereu, J.A. "Annuity Values Directly from Makeham Constants," Transactions of the Society of Actuaries vol. XIV (1962), pp. 269-286.

You are to simulate, using **Makeham**'s law, the future lifetime, T_{50}, of a person currently age 50.

(a) Use the result of simulating from **Gompertz** to suggest a procedure for simulating from **Makeham** using the acceptance-rejection method. Use the parameters given above. Give details of the procedure.

(b) Use this procedure to simulate 10,000 values of T_{50} from the **Makeham**'s law.

(c) Using these 10,000 simulated values, estimate the average future lifetime of a person currently age 50 and give a 95% confidence interval of this estimate. You may use the Central Limit Theorem (CLT) to produce the confidence interval.