Math 213/Roby **Practice Midterm** October 2007

1. Give the negation of the following statements, avoiding locutions like "It is not the case that…".

   (a) Fred goes bowling and says "Yabba-Dabba-Doo!"

   (b) If I take the test, I'll fail.

   (c) Every action has an equal and opposite reaction.

   (d) The product of any two odd numbers is prime.

2. For each of the following statements, indicate whether the statement is true or false. If true, give a short explanation of why (full proofs unnecessary). If false, provide a counterexample. Try to "salvage" the false statements by providing a similar (non-trivial) statement that would be true.

   (a) The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$ is onto.

   (b) Two functions $f, g : \mathbb{R} \to \mathbb{R}$ are bounded iff fg is bounded.

   (c) If two functions $f, g : \mathbb{R} \to \mathbb{R}$ are monotone then $g \circ f$ is monotone.

3. PODASIP: n is odd iff $n^2 - 1$ is divisible by 8.

4. Prove that for any sets A and B, $(A \cup B) \cap A^c = B - A$.

5. Let $\{a_n\}$ be a sequence with $a_1 = 1$ and $a_{n+1} = a_n + 3n(n+1)$ for $n \in \mathbb{N}$. Prove that $a_n = n^3 - n + 1$ for all $n \in \mathbb{N}$.

6. Let $f : A \to B$ and $g : B \to C$. Set $h = g \circ f$. PODASIP

   (a) if h is injective, then f is injective.

   (b) if h is injective, then g is injective.

   (c) if h is surjective, then f is surjective.

   (d) if h is surjective, then g is surjective.

7. Give an explicit bijection between $\mathbb{N} \times \mathbb{N}$ and $\mathbb{N}$.

8. Suppose you have infinitely many postage stamps in denominations 4 and 7, but no other denominations. Prove that you can create every integral amount of postage greater than 17 cents.

9. Go back over your old homework and quizzes to review and make sure you understand any problem on which you lost points. Compare them with the solutions handed out or done in class.