Name:______________________________ Section:____________

1. Complete the definitions:

(a) The column space of a matrix A is

(b) Let H be a subspace of a vector space V. An indexed set of vectors $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_p\}$ in V is a basis for H if

(c) The rank of a matrix A is

2. $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 5 & 4 & 0 \\ 3 & 1 & 6 & 5 \end{bmatrix}$

(a) Find A^T.

(b) Find $\det(A)$

(c) Is $\text{col}A = R^4$? Justify your answer.

3. $A = \begin{bmatrix} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 3 & 1 & 1 \\ 2 & 0 & 2 & 3 & -1 \\ 0 & 0 & 1 & -2 & -1 \\ 1 & 0 & 2 & 2 & 0 \end{bmatrix}$.

(a) Find a basis for the column space of B.

(b) Find a basis for $\text{span}\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 2 \\ 1 \\ 2 \end{bmatrix} \}$.

(c) What is the dimension of Nul B?

(d) Find a basis for the null space of B.

4. Let $H = \{ \begin{bmatrix} a + 2b + c + 2d \\ 3c + d \\ a + 2b - 2c + d \\ a + 2b + 10c + 5d \end{bmatrix} | a, b, c, d \text{ in } R\}$, a subspace of R^4.

(a) Show that the vector $\vec{x} = \{ \begin{bmatrix} -1 \\ 3 \\ -4 \\ 8 \end{bmatrix}$ is in H.

(b) Show that $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \\ 10 \end{bmatrix} \}$ is a basis for H.

(c) Find the coordinate vector $[\vec{x}]_{\mathcal{B}}$ of $\vec{x} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 8 \end{bmatrix}$ relative to the basis $\mathcal{B}$.