

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Pg 1 \_\_\_\_ Pg 2 \_\_\_\_ Pg 3 \_\_\_\_ Pg 4 \_\_\_\_ Pg 5 \_\_\_\_ Pg 6 \_\_\_\_ Pg 7 \_\_\_\_ Total \_\_\_\_

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. (12 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or describe a counterexample.

(a) If  $F(x) = \int_1^x f(t) dt$  and  $f(a) = 0$ , then  $a$  is a critical point of  $F$ .

(a) T F

Justification:

(b)  $y = xe^x$  is the solution to the differential equation  $\frac{dy}{dx} = y + e^x$  where  $y = 0$  when  $x = 0$ .

(b) T F

Justification:

(c) The Taylor series of  $\sin x$  about  $x = 1$  is  $(x - 1) - \frac{(x - 1)^3}{3} + \frac{(x - 1)^5}{5} - \dots$

(c) T F

Justification:

(d)  $2x \cos(x^2)$  is the derivative of  $\int_3^x 2t \cos(t^2) dt$ .

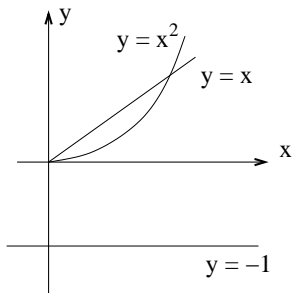
(d) T F

Justification:

2. (5 pts) Write  $e^{-i\pi/4}$  in the form  $a + ib$ .

3. (10 pts) Find the radius of convergence for the series  $\sum_{n=1}^{\infty} \frac{x^n}{3^n n^3}$ .

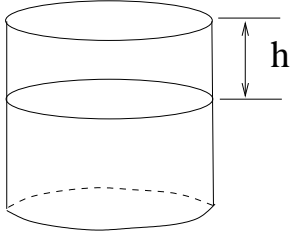
4. (10 pts) Consider the region bounded by  $y = x$  and  $y = x^2$ . Set up (but do not evaluate) a definite integral giving the volume of the solid obtained by rotating the region about the line  $y = -1$ .



5. (10 pts) Does the improper integral  $\int_3^{\infty} xe^{-x} dx$  converge or diverge? If convergent then show all steps in computing its value. If divergent, show why.
6. (10 pts) Evaluate  $\int_0^1 \frac{x+1}{x^2+1} dx$ , using techniques of integration learned in Math 116, and not calculator integration. Show all steps.

7. (10 pts) A cylindrical tank has a circular cross section of radius 2 meters and height 4 meters. It is filled with compressible liquid whose density varies with its depth. In particular the density  $\rho(h) = 60\sqrt{1+h}$  kg/m<sup>3</sup> at  $h$  meters below the surface.

a) What is the approximate mass in a thin slice of thickness  $\Delta h$  that is  $h$  meters below the top of the tank?



b) Write down a definite integral whose value is equal to the total mass of the liquid.

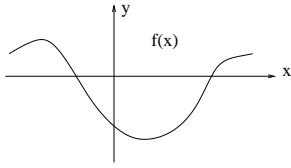
8. (5 pts) Find the slope of the line tangent to the parametric curve  $x = t \cos t$ ,  $y = 3t + t^5$  when  $t = 0$ .

9. (10 pts) A thermometer registered  $-20^{\circ}\text{C}$  outside and then was brought into the house where the temperature was  $24^{\circ}\text{C}$ . After 5 minutes, it registered  $0^{\circ}\text{C}$ . According to Newton's Law of Heating and Cooling, the rate of change of temperature of an object is proportional to the difference between temperature of the object and the temperature of the surrounding air.

(a) When will it register  $20^{\circ}\text{C}$ ?

(b) Using the work from (a), discuss the thermometer reading over the time interval  $[0, \infty)$ .

10. (5 pts) The graph of a function  $f(x)$  is shown below (graph will show  $f(0) < 0$ ,  $f'(0) < 0$ ,  $f''(0) > 0$ ). Which of the following could be the Taylor polynomial approximating  $f(x)$  for  $x$  near 0? More than one answer is possible. Justify your answer.



- (a)  $P_2(x) = 2 + 2x + 2x^2$       (b)  $P_2(x) = 2 - 2x + 2x^2$       (c)  $P_2(x) = 2 + 2x - 2x^2$   
 (d)  $P_2(x) = 2 - 2x - 2x^2$       (e)  $P_2(x) = -2 + 2x + 2x^2$       (f)  $P_2(x) = -2 - 2x + 2x^2$   
 (h)  $P_2(x) = -2 - 2x + 5x^2$       (i)  $P_2(x) = -2 - 2x - 2x^2$       (j)  $P_2(x) = -2 + 2x - 2x^2$

11. (15 pts) Determine which of the following series converges or diverges. Circle CONVERGES or DIVERGES and then briefly explain WHY each series converges or diverges. For each series, your explanation is worth 3 points while circling the correct answer is worth 2 point.

(a)  $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$       CONVERGES    DIVERGES

EXPLANATION:

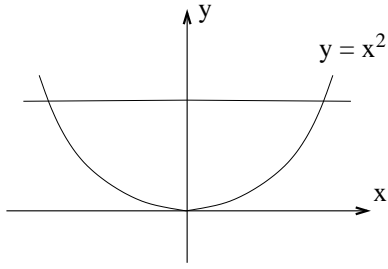
(b)  $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + 2}}$       CONVERGES    DIVERGES

EXPLANATION:

(c)  $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$       CONVERGES    DIVERGES

EXPLANATION:

12. (10 pts) A solid figure P has a base region  $\{(x, y) | x^2 \leq y \leq 1\}$  in the  $xy$ -plane. Every cross-section of P by a plane perpendicular to the  $x$ -axis is a square. Find the volume of P.



13. (10 pts) Use separation of variables to solve the differential equation  $\frac{dy}{dt} = y^2(2t + 1)$  so that  $y = 1$  when  $t = 0$ .