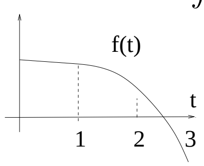
Name:______________________________ Section:____________

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, provide a short explanation or counterexample.

(a) If $F(x) = \int_x^1 f(t)\,dt$ (see the graph of f) then $F(x)$ is concave down for all x between 1 and 2.



(a) T F

Justification:

(b) If $F(x) = \int_1^x f(t)\,dt$ and $f(a) = 0$, then a is a critical point of F. (b) T F

Justification:

(c) If $F(x) = \int_1^x f(t)\,dt$, $f'(a) = 0$ and $f''(a) > 0$, then a is an inflection point of F. (c) T F

Justification:

(d) If $F(x) = \int_1^x \frac{e^t}{t}\,dt$, then $\int F(x)\,dx = x\,F(x) - e^x + C$. (d) T F

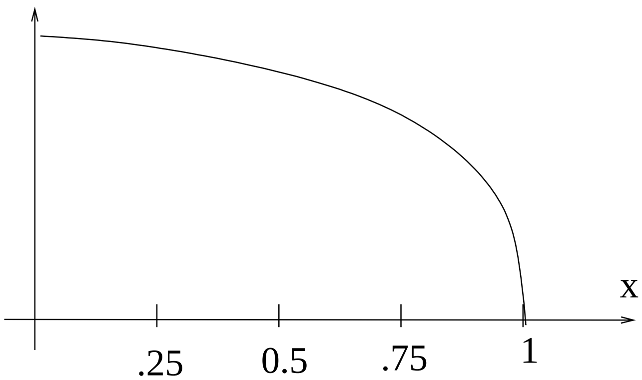
Justification:

2. It is desired to evaluate the definite integral $\int_0^1 (1 - e^{-x})\, dx$ using numerical methods. A graph of $f(x) = (1 - e^{-x})$ from $x = 0$ to $x = 1$ is shown. Here is a table of values of $f(x)$ that you can use:

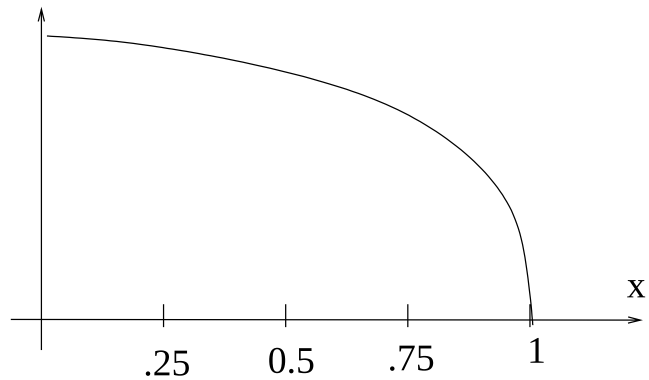
x	0	.25	0.5	.75	1
$f(x)$	0	.2212	.3935	.5276	.6321

(a) Using two divisions of the interval $[0, 1]$, find the left, right, trapezoid and midpoint approximations to $\int_0^1 (1 - e^{-x})\, dx$.

(b) Draw sketches showing what each approximation represents. State whether each approximation is an overestimate or an underestimate of the integral and give an appropriate explanation.



LEFT(2) & RIGHT(2)



MID(2) & TRAP(2)

	Approx.	over/under	Explanation
MID(2)			
TRAP(2)			

3. Do the following improper integrals converge? If so, calculate the value. If not, explain why not.

(a) $\displaystyle\int_{-1}^{0} \frac{1}{\sqrt{x+1}}\, dx$

(b) $\displaystyle\int_{0}^{\infty} x\, e^{-x}\, dx$

4. Does the following improper integral converge? Do not calculate the value but give an argument to support your answer.

$$\int_{0}^{1} \frac{\sin^2 x}{(x-1)^2})\, dx$$

5. Let b be a positive number. The region bounded by the positive y-axis, the positive x-axis, the vertical line $x = b$ and the curve $y = e^{-2x}$ is revolved about the x-axis.

(a) Find the volume of the resulting solid.

(b) Suppose we let $b \longrightarrow \infty$, creating a solid with an infinitely long neck. Does this solid have finite volume? If so, find it. If not, explain why not.

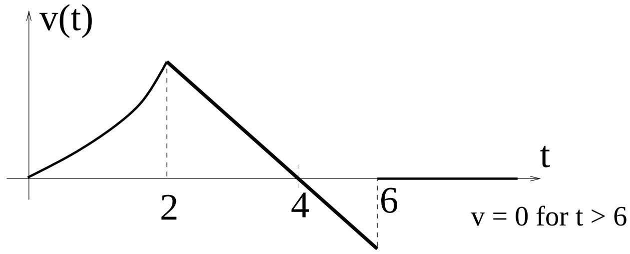
6. A solid figure P has as a base the region $\{(x, y)|x^2 \leq y \leq 1\}$ in the xy-plane. Moreover, every cross-section of P by a plane perpendicular to the x-axis is an equilateral triangle. Find the volume of P.

7. Find the exact area of the region under $y = x\sin(2x)$ and above the x-axis for $0 \le x \le \pi/2$.

8. Find the integral, showing all steps: $\displaystyle\int \frac{1}{(9-x^2)^{3/2}}\,dx$.

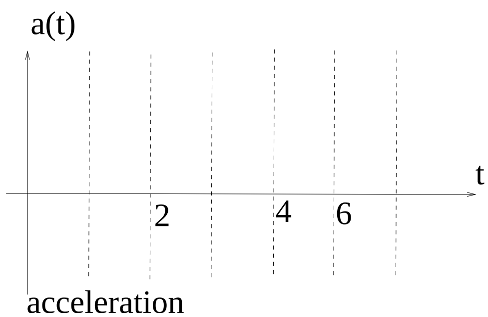
9. Find the integral : $\displaystyle\int \frac{2x^3+x^2+2x-1}{x^4-x^2}\,dx$.

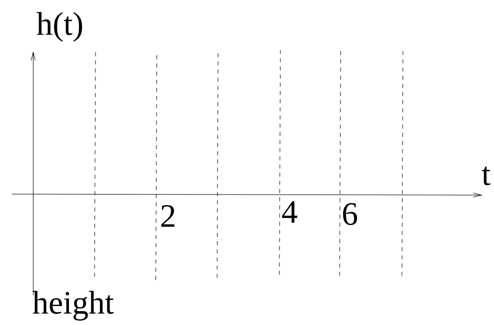
A young girl, who aspires to be a rocket scientist, launches a model rocket from the ground at time $t = 0$. The rocket travels straight up in the air, and the following graph shows the upward velocity of the rocket as a function of time.



Let $h(t)$ be the height (or vertical displacement) of the rocket, let $a(t)$ be the acceleration of the rocket, and let $h(0) = 0$ be the ground level from which the rocket was launched.

(a) Sketch a graph of the acceleration $a(t)$ of the rocket as a function of time.
(b) Sketch a graph of the height $h(t)$ of the rocket as a function of time.





(c) Let $v(t)$ denote the velocity of the rocket at time t. On which time interval, $0 \leq t \leq 2$ or $2 \leq t \leq 4$, does the rocket travel the greater distance?

(d) What is the sign of $\int_4^6 v(t)\, dt$? What does this mean physically?

(e) Describe the height and velocity of the rocket when $t > 6$. Write a plausible story about what happened to the rocket.