

Name: _____

Section: _____

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. No explanation is necessary.

(a) (3 pts) A local maximum of a function is always a critical point. (a) T F

(b) (3 pts) If $\int_a^b f(x)dx = 0$ implies that f is the constant zero function. (b) T F

(c) (3 pts) According to L'Hopital's rule, $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = -\frac{1}{6}$. (c) T F

(d) (3 pts) If $f''(x) = 0$ at $x = 0$, then the graph of f changes concavity at $x = 0$. (d) T F

(e) (3 pts) If a function is not continuous at a point, then the function is not defined at that point.

(e) T F

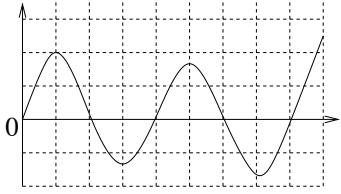
2. (7 pts) An object is traveling along a line. Its velocity is described by the function $v(t) = \sqrt{1 + t^3}$, where its units are meters per seconds. In order to approximate the distance traveled by the objects in the first 2 seconds within 0.01 meters, how often does one have to record the velocity of the object?

3. Let $f(x) = \sqrt{x + 1}$.

(a) (5 pts) Find the equation of the tangent line to the graph of $f(x)$ at $x = 0$.

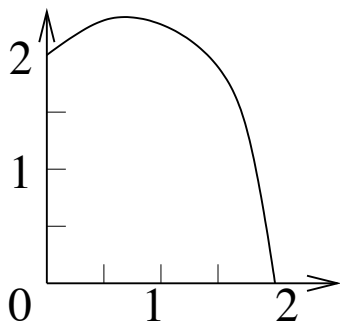
(b) (3 pts) Use the tangent line approximation to estimate $\sqrt{0.9}$.

4. The figure below is the graph of the **derivative** $f'(x)$ of a function $f(x)$. The unit on the x -axis is 1 and on the y -axis is 10.

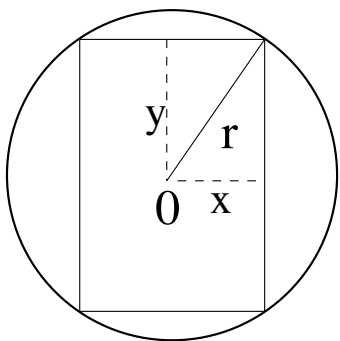


- a) (5 pts) At which x values (roughly) does f have local extrema? Describe the nature of the extrema (local max or min) and explain your answers briefly.
- b) (4 pts) In which interval(s) is f concave-up. Explain your answers briefly.
- c) (3 pts) At which point(s) does f have inflection points? Explain your answers briefly.
- d) (4 pts) Assuming that $f(0) = 0$, is it true or false that the global max of f in $[0,9]$ is likely to be a positive number? Explain your answer briefly.
- e) (4 pts) Assuming that $f(0) = 0$, is it true or false that the global min of f in $[0,9]$ is likely to be a positive number? Explain your answer briefly.
5. (5 pts) A spherical snow ball is melting. It radius decreases at a constant rate of 3 cm per minute from an initial value of 100 cm. How fast is the volume decreasing 10 minutes later? The volume is given by $V = 4/3\pi r^3$.

6. Below is a graph of the function $f(x) = -x^2 + x + 2$ on the interval $[0, 2]$.



- (a) (5 pts) Approximate the area between the curve and the x -axis on the interval $[0, 2]$ by using the left Riemann sum with $n = 4$ subintervals.
- (b) (2 pts) On the graph, draw the rectangles which correspond to the calculation in (a).
- (c) (3 pts) Use the Fundamental Theorem of Calculus to calculate the area of the region exactly. Show your work.
7. (10 pts) What are the dimensions of a rectangle that is inscribed to a circle of a constant radius r and that maximizes the area among all such inscribed rectangles? One may proceed as follows. First, find a relation among x, y, r . Next, find the formula expressing the area of such a rectangle in x, y . Then eliminate one of the variables and use the first derivative test. Show your work.



8. Suppose that a function f define in $[0,2]$ has $\sqrt{4 + 2x + x^3}$ as an anti-derivative.
- a) (3 pts) Find $\int_0^2 f dx$. No approximation is accepted.
- b) (2 pts) What is the average value of f in $[0,2]$?
9. A mouse moves back and forth in a straight tunnel. The graph of the mouse's velocity function $v(t)$ (feet/seconds) is given in the figure below. The velocity is positive when the mouse moves right and negative when it moves left. Assume that the mouse is at the center of the tunnel when $t = 0$.
- a) (5 pts) At which time(s) does the mouse change direction? Explain your answer briefly.
- b) (5 pts) At which time(s) does the mouse move most rapidly to the right? Explain your answer briefly.
- c) (5 pts) Using the definite integral, express the farthest distance the mouse reached to the right of the center in the first 10 seconds. You need not calculate the integral. Explain your answer briefly.
- d) (5 pts) Using the the definite integral, express the total distance the mouse traveled in the first 10 second. You need not calculate the integral. Explain your answer briefly.

