MATH 115Q – FALL 2005 –FINAL EXAM

December 15, 2005

Time limit: 120 minutes

Name:______________________________ Instructor:______________________________

Please read all instructions and problems **carefully**.

You may not consult any books or papers. You may use a calculator and your 3" $\times$ 5" notecard.

There are two extra blank pages at the end of the exam. You may use these for computations, but **we will not read them.** Please transfer all final answers to the page on which the question is posed.

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Problem:	Your score:	Total points
1		12 points
2		7 points
3		12 points
4		10 points
5		6 points
6		6 points
7		12 points
8		13 points
9		13 points
10		9 points
Total:		100 points

1. [12 pts.] In 1910 the population of Mars was about 12 million and growing at 3.5% annually.

(a) Write an expression for the function $f(t)$ that gives the population of Mars, in millions, t years after 1910.

Final answer to (a):

$f(t) =$

(b) Find the rate of growth, in people/year, of the population of Mars in 1915.

Final answer to (b):

(c) Interpret the statement $(f^{-1})'(30) = 0.96895$ in practical terms.

2. [7 pts.] The differentiable function g has the following properties: $g(2) = 5,\ g'(2) = 3,\ g(0) = -2,\ g'(0) = 4$. Show your work below.

(a) If $f(x) = \ln(g(x)),$ find $f'(2)$.

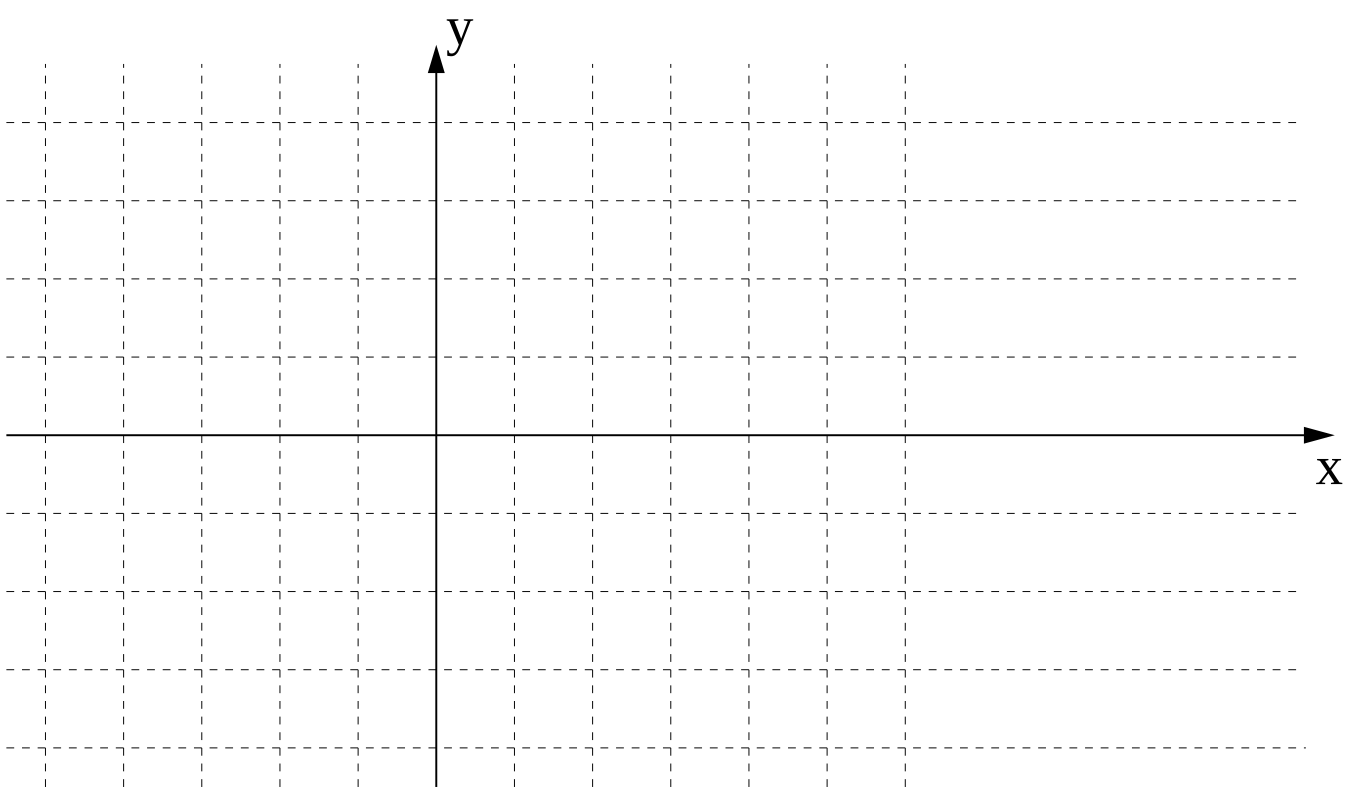
Final answer to (a):
$f'(2) =$

(b) If $h(x) = \dfrac{g(x)}{x^2 + 1},$ find $h'(0)$.

Final answer to (b):
$h'(0) =$

3. [12 pts.] Sketch the graph of the function f with the following properties.
Label significant points and lines.

- f is defined and differentiable for all real numbers.
- $f(-1) = 1$, f has a global maximum value of 4 and f has no global minimum
- $f'(x) > 0$ for x in the intervals $(-\infty, -3)$ and $(0, \infty)$
- $f'(x) < 0$ for x in the interval $(-3, 0)$.
- $f''(x) > 0$ for x in the interval $(-1, 1)$
- $f''(x) < 0$ for x in the intervals $(-\infty, -1)$ and $(1, \infty)$.
- $\lim_{x\to\infty} f(x) = 2$.



4. [10 pts.] The volume of a right circular cylinder of height h and radius r is $V = \pi r^2 h$. A computer program allows you to manipulate the shape of a cylinder by varying its radius and height independently. You set the program so that the cylinder's height is increasing at a rate of 1 cm/sec and the radius is decreasing at a rate of 0.04 cm/sec. At what rate is the volume changing when the height of the cylinder is 12 cm and the radius is 1 cm? Indicate units. (Notice that in this problem both h and r are functions of t.)

Final answer:

5. [6 pts.] Find the slope of the curve $xy + y^2 = 1$ at the point (0,1).

Final answer:

6. [6 pts.] Consider the table of values for the function f:

x	0	20	40	60	80	100
$f(x)$	1.2	2.8	4.0	3.2	5.1	5.2

Give upper and lower estimates to the value of the integral $\int_0^{100} f(x)\,dx$. Show your work.

lower estimate:

upper estimate:

7. [12 pts.] The functions f and g are continuous, f is even, g is odd, $\int_0^1 f(x)\,dx = 5$ and $\int_0^1 g(x)\,dx = 7$. In each part below compute the integral or, if you do not have enough information to do so, write "not enough information." Show your work.

(a) $\int_0^1 (f(x) - g(x))\,dx$.

Final answer to (a):

(b) $\int_0^1 3g(x)\,dx$.

Final answer to (b):

(c) $\int_0^1 f(x)g(x)\,dx$.

Final answer to (c):

(d) $\int_{-1}^1 (f(x) + g(x))\,dx$.

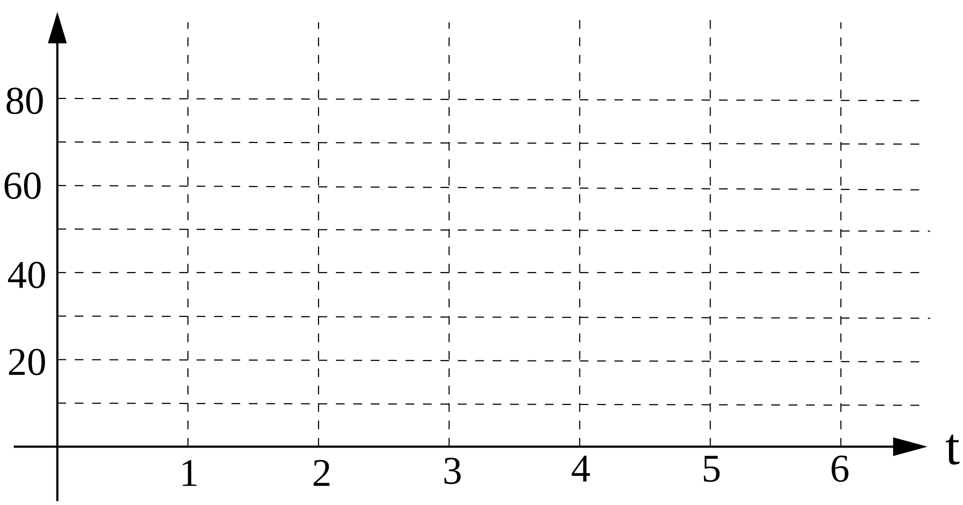
Final answer to (d):

8. [13 pts.] A large tank is being filled with fresh drinking water. The water flows into the tank at a rate of $r(t) = 50 + 10\cos(\frac{\pi}{2}t)$ gallons/hour, where t denotes the number of hours elapsed since we began watching the tank fill.

(a) How fast is the rate of flow changing when $t = 1$? Include appropriate units.

Final answer to (a):

(b) Sketch the graph of $r(t)$ on the axes.



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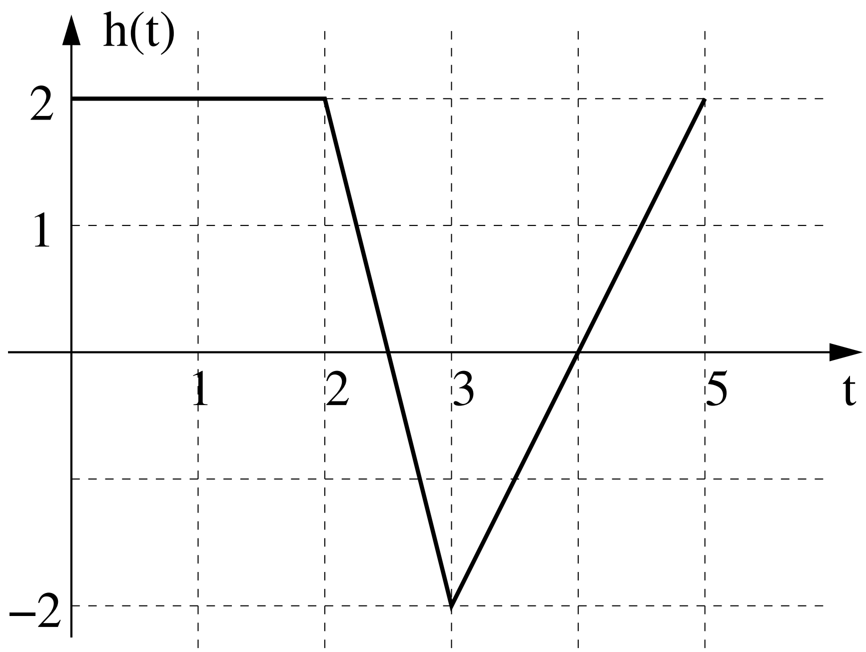
(c) Use the definite integral to express the area of the region under the graph of $r(t)$, above the t-axis and between the lines $t = 0$ and $t = 3$. (Leave your answer as a definite integral. Do not evaluate it.)

Final answer to (c):

(d) What is the practical meaning of the integral in (c) in terms of the water tank?

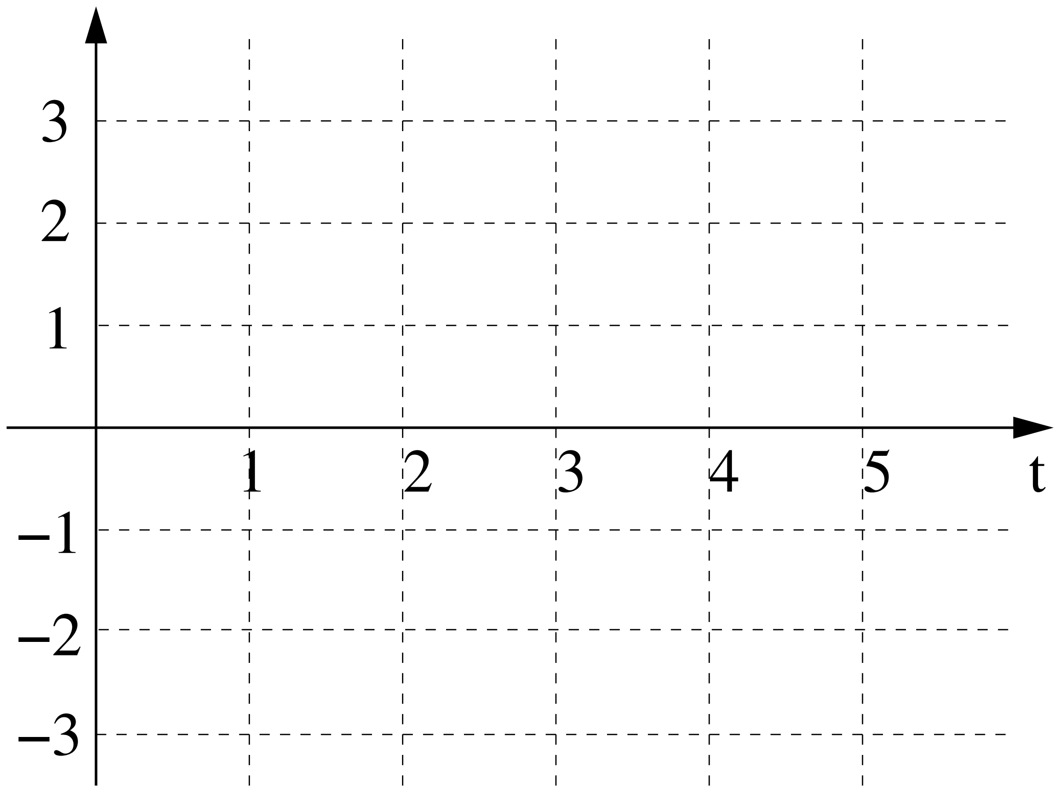
9. [13 pts.] The graph of the function h is given below.



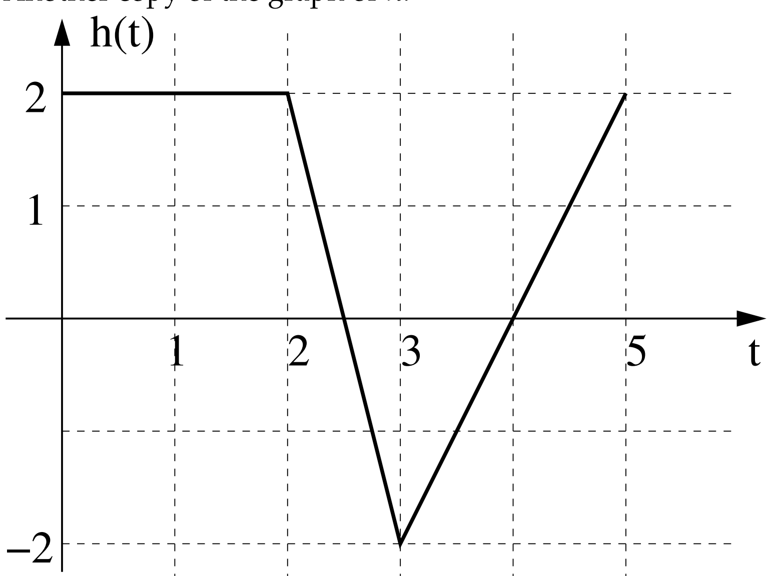
(a) Complete the table

b	0	2	3	4	5
$\int_0^b h(t)\,dt$					

(b) $G(t)$ is a function whose derivative is $h(t)$, ($G'(t) = h(t)$). Sketch the graph of G, assuming that $G(0) = -2$. Be as accurate as you can possibly be using the information given. Include the usual things like critical points, inflection points, concavity, etc. and, where possible, their coordinates.



Another copy of the graph of h:



10. [9 pts.]

(a) Find the number c for which $\int_1^c \frac{6}{x^2}\,dx = 2$. Show all work.

Final answer to (a):

(b) Evaluate the indefinite integral $\int (1 + \sin x - 2e^x)\,dx$.

Final answer to (b):

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