



MATH 115Q – FALL 2005 – MIDTERM EXAM II

November 15, 2005

Time limit: 120 minutes

Name: _____

Instructor: _____

Please read all instructions and problems **carefully**.

You may not consult any books or papers. You may use a calculator and your 3" × 5" notecard.

There are two extra blank pages at the end of the exam. You may use these for computations, but **we will not read them**. Please transfer all final answers to the page on which the question is posed.

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Problem:	Your score:	Total points
1		10 points
2		10 points
3		12 points
4		12 points
5		10 points
6		16 points
7		10 points
8		10 points
9		10 points
Total:		100 points

1. [10 points] Suppose that a function f satisfies $f(1) = 3$ and its **DERIVATIVE** is

$$\frac{d}{dx} \left(f(x) \right) = e^{-x^2}.$$

In (a) and (b), please show your work, and write your final answer in the box provided.

(a) Find the local linearization of $f(x)$ near $x = 1$.

Final answer to (a):

(b) Estimate the value of $f(1.5)$.

Final answer to (b):

$f(1.5) \approx$

(c) Is your answer in (b) an under- or an over-estimate? Please give an argument supporting your answer.

2. [10 points] Suppose that $f(x) = xe^x$. Show your work on the following problems, and write your final answer in the box provided.

(a) Determine the value of $f^{-1}(e)$?

Final answer to (a):

$$f^{-1}(e) =$$

(b) Determine the value of $(f^{-1})'(e)$?

Final answer to (b):

$$(f^{-1})'(e) =$$

3. [12 points] Suppose y is defined implicitly by the relation $\sin(2x) - \cos(3y^2) = 1$.

(a) Find $\frac{dy}{dx}$. Show your work, and put your final answer in the box below.

Final answer to (a):

$$\frac{dy}{dx} =$$

(b) What is the equation of the tangent line to the curve $\sin(2x) - \cos(3y^2) = 1$ at the point $(\frac{\pi}{4}, \sqrt{\frac{\pi}{6}})$? Show your work, and put your final answer in the box below.

Final answer to (b):

(c) Are there points on the curve $\sin(2x) - \cos(3y^2) = 1$ where the tangent lines are vertical? Explain your answer.

4. [12 points] Please **CIRCLE** True or False for each statement. If the statement is False, then give an example contradicting the statement; if the statement is true, then state a relevant theorem that guarantees it is true.

(a) If f is continuous for $0 < x < 1$ and achieves a local maximum at $x_0 = \frac{1}{2}$, then $f'(\frac{1}{2}) = 0$.

TRUE	FALSE
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JUSTIFICATION:

(b) If f is differentiable for $-2 < x < 2$ and $f(-1) = f(1)$, then there exists a value c with $-1 < c < 1$ such that $f'(c) = 0$.

TRUE	FALSE
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JUSTIFICATION:

(c) If $f'(x) > 0$ for all x , then f is an increasing function.

TRUE	FALSE
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JUSTIFICATION:

(d) If a differentiable function f is defined at c , and c is a critical point of f , then f has either a local maximum or a local minimum at c .

TRUE	FALSE
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JUSTIFICATION:

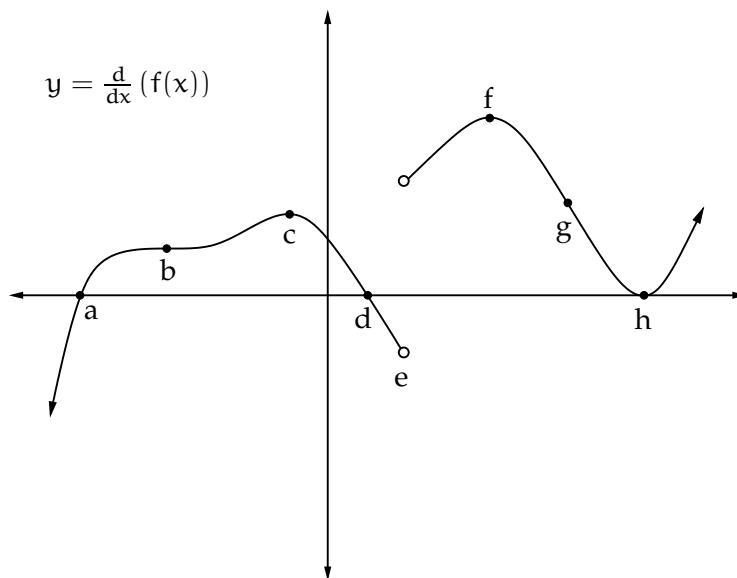
(e) If f is a differentiable function and $f(0) < f(1)$, then there is some $0 < c < 1$ with $f'(c) > 0$.

JUSTIFICATION:

TRUE	FALSE
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5. [10 points] Give an argument verifying that $e^x \geq x + 1$ for all $x \geq 0$.

6. [16 points] The graph below shows the **DERIVATIVE** $\frac{d}{dx}f(x)$ of a continuous function $f(x)$. Using this, determine whether the marked points correspond to critical points, inflection points, and local maxima or minima of the original function $y = f(x)$. For those points that **are** critical, etc., mark a “Yes” in the following table; otherwise, leave the space blank.



The graph of the derivative of $f(x)$.

Point	Critical for $f(x)$?	Inflection for $f(x)$?	Local max for $f(x)$?	Local min for $f(x)$?
a				
b				
c				
d				
e				
f				
g				
h				

7. [10 points] Please note that the formula for the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$, and the formula for the surface area of a sphere of radius r is $A = 4\pi r^2$. A clown is filling a balloon with helium from a tank that emits helium at a rate of $250\text{cm}^3/\text{sec}$.

Please answer the following questions, specifying the **units of your answers**. Show your work, and put your final answer in the box provided.

- (a) How fast is the radius of the balloon changing when the radius is 5cm?

Final answer to (a):

- (b) How fast is the surface area of the balloon changing when the radius of the balloon is 5cm?

Final answer to (b):

- (c) After 20 seconds, the balloon escapes the clown's grip. Helium blows out of the balloon at a rate of $350\text{cm}^3/\text{sec}$. How fast is the radius changing when the radius of the balloon is 10cm?

Final answer to (c):

8. [10 points] Consider the continuous function

$$f(x) = x^3 + 3x^2 - 24x - 4$$

on the closed interval $-4 \leq x \leq 4$.

- (a) At what values of x does f have a local minimum? A local maximum? Please show your work and give exact answers in the box provided.

Final answer to (a):

- (b) Does f achieve a global maximum and a global minimum on this interval? State why or why not. If they do exist, what are their values?

9. [10 points] A team of Navy SEALs is located 20km west and 6km south of their inland extraction point. They are moving east in their Humvee at 25km/hr along a beach that runs east-west. Once they leave the beach, they must abandon their vehicle. Intelligence estimates that they will only be able to move at 12km/hr over the land terrain.

Please answer the following questions, showing your work and marking your final answer in the box.

- (a) How far down the beach from their current position should the SEAL team leave the vehicle and head inland in order to reach the extraction point in the shortest possible time?

Final answer to (a):

- (b) If the current time is 01 : 35 and the extraction is scheduled to occur at exactly 02 : 50, will they make it in time?

Final answer to (b):

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