

Name: _____

Section: _____

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F.

(a) (2 points) A global maximum is always a critical point. (a) T F

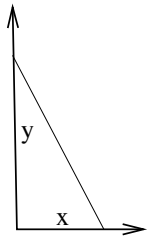
(b) (2 points) A function defined on all points of a closed interval always has a global maximum and a global minimum. (b) T F

(c) (2 points) The inflection points of a function f are the local extrema of f' . (c) T F

(d) (2 points) If $f''(x) = 0$ at $x = 0$, then the graph of f changes concavity at $x = 0$. (d) T F

(e) (2 points) If $f' > 0$ on an interval, the function is concave up on the interval. (e) T F

2. The foot of the ladder of length 10 feet, shown in the figure, moves away from the wall at a speed of 2 ft/sec, causing the top of the ladder to slide down the wall without leaving it.



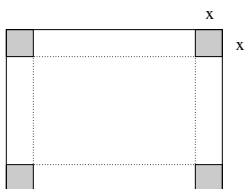
(a) (4 points) Find the equation expressing the relation between $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

(b) (4 points) Is it true or false that $\frac{dx}{dt}$ and $\frac{dy}{dt}$ have the same sign? Justify your answer.

(b) (4 points) Is it true or false the top of the ladder is moving faster and faster? Justify your answer.

(c) (4 points) Keeping $\frac{dx}{dt}$ constant, doubling x and y and the length of the ladder, doubles $\frac{dy}{dt}$. Is this statement true or false? Justify your answer.

3. (10 points) The volume of a certain tree is given by $V = \frac{1}{12\pi}C^2h$, where C is the circumference of the tree at the ground level and h is the height of the tree. If C is 5 feet and growing at the rate of 0.2 feet per year, and if h is 22 feet and is growing at 4 feet per year, find the rate of growth of the volume V .
4. Let g be a function such that $g(2) = 4$ and whose derivative is known to be $g'(x) = \sqrt{x^2 + 1}$.
- (a) (5 points) Use a linear approximation to estimate the value of $g(1.95)$. Show your work.
- (b) (5 points) Do you think your estimate in part (a) is an overestimate or an underestimate? Explain.
5. (10 points) An open top rectangular box is to be made from a 12×18 inch piece of cardboard by cutting a square of side x from each corner and folding up the sides along the dotted edges (See figure below). Find x at which the resulting box has the (global) maximum volume.



6. Suppose two functions $f(x) = e^x$ and $g(x) = x + 1$ are given for all values of x .

(a) (4 points) Explain graphically why $f(x) \geq g(x)$ for all x .

(b) (4 points) Prove $f(x) \geq g(x)$ for all x using the racetrack principle.

7. Below is the graph of $f(x)$, the derivative of $F(x)$ (that is, $F'(x) = f(x)$). Answer the following:

(a) (2 points) What are the interval(s) on which $F(x)$ is increasing?

(b) (2 points) What are the interval(s) on which $F(x)$ is concave down?

(c) (2 points) What value(s) of x will give a local maximum for $F(x)$?

(d) (2 points) Suppose $F(0) = 2$. Determine the value of $F(4)$.

8. The table gives values for functions f and g , and their derivatives. Determine the given derivatives and write down the formulas used.

x	0	1	2
$f(x)$	5	-1	4
$g(x)$	2	2	0
$f'(x)$	-2	2	3
$g'(x)$	5	6	0.5

(a) (2 points) If $K(x) = \frac{f(x)}{g(x)}$ then $K'(0) = \underline{\hspace{2cm}}$?

(b) (2 points) If $H(x) = f(g(x))$ then $H'(1) = \underline{\hspace{2cm}}$?

(c) (2 points) If $F(x) = e^{f(x)}$ then $F'(1) = \underline{\hspace{2cm}}$?

(d) (2 points) If $G(x) = (f(x))^2$ then $G'(1) = \underline{\hspace{2cm}}$?

(c) (2 points) If $K(x) = \sin(g(x))$ then $K'(2) = \underline{\hspace{2cm}}$?

9. A function $f(x) = x^3 - 6x + 1$ is given in $[-2, 3]$.
- a) (7 points) Find the local extrema (maxima and minima) of f using the first derivative test. Clearly indicate where these local extrema occur and what values they are.
- b) (3 points) Find the global (absolute) extrema of f . Clearly indicate where these global extrema occur and what values they are.
- c) (3 points) Find the inflection points if any.
- d) (5 points) Sketch the graph of f . Make sure all pertinent pieces of information such as local and global extrema, y intercept, inflection points, concavity etc to be indicated.