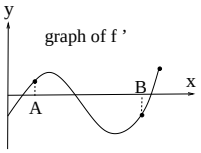
Name:_____ Section:_____

Directions: Please read each question carefully. Solution methods must be complete, logical and understandable, answers must be clearly labeled and explanations must be clearly written in the space provided. Calculators are allowed but you must show all your work to receive full credit on a problem.

1. (10 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a **complete sentence** justifying your answer.

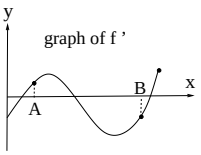
(a) Given the graph of f' as shown, f is increasing at the point B.



(a) T F

Write a sentence justifying your answer:

(b) Given the graph of f' as shown, f is concave up at the point A.



(a) T F

Write a sentence justifying your answer:

(c) If $f'(x) > 0$ for all x in the interval (a,b), then f is increasing on the interval (a,b). (c) T F

Write a sentence justifying your answer:

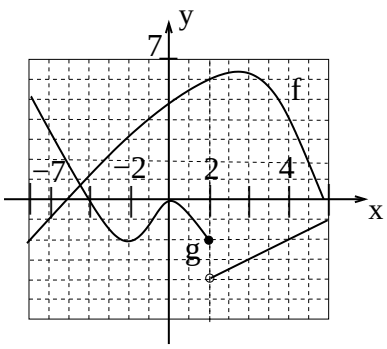
(d) If a function is not continuous at a point, then it is not defined at that point. (d) T F

Write a sentence justifying your answer:

(e) If a function f is decreasing on an interval, then f' is decreasing on that interval. (e) T F

Write a sentence justifying your answer:

2. (10 pts) The graphs of two functions, f and g are given in the figure below. Use the graphs to estimate each of the following:



Each box is 1 unit

(a) $f(g(-2))$ _____

(b) $\lim_{x \to 4^+} (f(x) + g(x))$ _____

(c) $f'(-7)$ _____

(d) The sign of $f''(3)$ is _____

3. (10 pts) Suppose $f(t) = 1.8e^{0.248t}$ is the population of Mexico in millions, where t is the number of years since 1980.

(a) What is the average rate of change of the population between 1992 and 1996?

(b) Explain the practical meaning of the statement $f(12) = 35.30$

(c) Explain the practical meaning of the statement $f^{-1}(95.18) = 16$

(d) What are the units for $f'(t)$?

(e) Explain the practical meaning of the statement $f'(12) = 8.75$

4. (10 pts) $f(x) = \begin{cases} 8 - x & \text{for} \quad x < 4 \\ x^2 - x & \text{for} \quad x \geq 4 \end{cases}$

(a) Determine $f(4)$

(b) Find $\lim_{x \to 4^+} f(x)$

(c) Complete the definition (using limits): f is **continuous** at $x = a$ if

(d) Using the definition in (c), explain carefully why $f(x)$ is not continuous at $x = 4$.

5. (5 pts) The depth of water at a particular point in a bay is given by the function $y = 20 + 8\cos(t/2)$, where y is in feet and t is time in hours since midnight.

(a) What is the amplitude?

(b) Find the period for this function..

(c) What is the depth of the water at low tide?

(d) Give one time when tide occurs.

6. (10 pts) An exponentially decaying substance was weighed every hour and the results recorded in one of the tables below. Unfortunately, there was a mix up with a second linear table which is also given below.

Time	Weight (in grams)
9 am	10.000
10 am	8.958
11 am	7.916
12 noon	6.874
1 pm	5.832

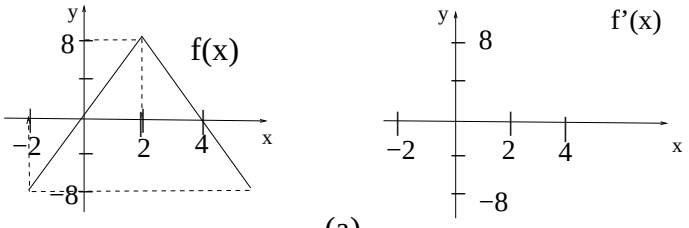
Time	Weight (in grams)
9 am	10.000
10 am	8.958
11 am	8.025
12 noon	7.189
1 pm	6.440

(a) Determine which one is exponential and find a formula of the form $Q(t) = Q_0 e^{-kt}$ which would give the weight of the substance $Q(t)$ at time t in hours since 9 am.

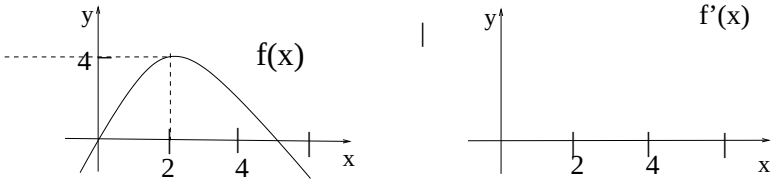
(b) Explain your reasoning in part (a).

(c) How long will it be until 80% of the original amount has decayed?

7. (10 pts) Sketch possible graphs of $f'(x)$ for the functions given:



(a)



(b)

8. (10 pts) (a) Use the definition of the derivative and algebra to evaluate the derivative of the function $f(x) = 1/x$ at $x = 3$.

(b) Find the equation of the tangent line to $y = 1/x$ at $x = 3$.

9. (10 pts) Let $f'(x) = 500(x - 1)(3 - x)$ (NOTE: This is the derivative of f).

(a) Determine the intervals on which f is increasing.

(b) Find $f''(x)$.

(c) Determine the inflection points of $f(x)$.

10. (10 pts) A particle moves in such a way that its position along the y-axis is given by $y(t) = 3t^2 - 8\dfrac{1}{\sqrt{t}}$ over the time interval $1 \leq t \leq 10$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}t}$:

(b) What is the velocity at $t = 4$?

(c) What is the acceleraton at $t = 4$?

(d) When is the velocity decreasing (find all times t)?

11. (5 pts) The table below gives the values of a function $f(x)$ at various x.

x	0	1	2	3	4
$f(x)$	0.5	0.580	0.912	1.691	3.218

a) What is the average rate of change in the interval $[1, 3]$?

b) What is possibly the most appropriate approximation of $f'(2)$? Justify your answer

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