

1. (a) [5] Give the linear approximation  $L(x)$  of the function  $f(x) = (1 + x)^{12}$  at  $a = 0$ .  
 (b) [5] Use the linear approximation to estimate  $(1.002)^{12}$ .
2. [15] What are the absolute maximum and minimum values of the function  $g(x) = x - x^{1/3}$  on the interval  $[-2, 8]$ ?
3. [20] Sketch the graph of the function  $f$  with the following properties. Label significant points and lines.
  - $f$  is defined and differentiable for all real numbers.
  - $f(0) = 0$ ,  $f'(0) = 0$  and  $f(2) = -3$ .
  - $f'(x) > 0$  for  $x$  in the interval  $(2, \infty)$ .
  - $f'(x) < 0$  for  $x$  in the interval  $(-\infty, 2)$ .
  - $f''(x) > 0$  for  $x$  in the intervals  $(-\infty, 0)$  and  $(1, 3)$
  - $f''(x) < 0$  for  $x$  in the intervals  $(0, 1)$  and  $(3, \infty)$ .
4. (a) [5] State the Mean Value Theorem.  
 (b) [10] Verify that the conclusion of the Mean Value Theorem holds for the function  $f(x) = 2x^2 - 1$  on the interval  $[1, 2]$ .
5. Find the intervals on which the graph of the function  $f(x) = 2x^4 - 4x^2 + 3$  is
  - (a) [5] Increasing \_\_\_\_\_  
 Decreasing \_\_\_\_\_
  - (b) [5] Concave up \_\_\_\_\_  
 Concave down \_\_\_\_\_
  - (c) [3] Locate points at which the graph of  $f$  has local extrema(max or mins) \_\_\_\_\_
  - (d) [2] Find any inflection points \_\_\_\_\_
6. [10] A rectangle has its base on the x-axis and its upper two vertices on the parabola  $y = 27 - x^2$ . What is the largest area the rectangle can have and what are its dimensions?
7. (6 pts) A function  $f$  has the following properties:  $f(0) = 7$  and  $f'(0) = -9$ . If  $G(x) = f(3x^5 + 2x)$  find  $G'(0)$ .
8. (10 pts) Find the line tangent to the graph of  $y^4 - 3 = -x^2y^2$  at the point  $(\sqrt{2}, 1)$ .

9. (10 pts) A cannon ball is shot straight up into the sky so that its height in feet at time  $t$  is given by  $s = f(t) = -32t^2 + 128t + 5$ . How high does the cannon ball go before it falls back down?
10. (10 pts) A stone dropped into a pool of water creates a ripple which has the shape of an expanding circle. If the radius of the circle increases at a rate of 3 feet per second, find how fast the area of the circle is increasing after 5 seconds.
11. A manufacturer wants to design a box with an open top, a square base and a surface area of  $108 \text{ in}^2$ . What dimensions will produce a box with the maximum volume?
12. [10]A girl flies a kite at a height of 300 ft. The wind carries the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out string when the kite is 500 ft away from her?
13. Does  $f(x) = x^4 + 1$  have an inflection point? Explain.