

Name: \_\_\_\_\_

Section: \_\_\_\_\_

**Directions:** Please read each question carefully. Solution methods must be complete, logical and understandable, answers must be clearly labeled and explanations must be clearly written in the space provided. Calculators are allowed but you must show all your work to receive full credit on a problem.

1. (10 pts)

(a) State the formal definition of  $\lim_{x \rightarrow c} f(x) = L$ .

(b) Briefly explain the meaning of your definition in practical terms:

2. (10 pts)

(a) State the formal definition of  $f'(a)$ , the *derivative* of  $f$  at the point  $x = a$ .(b) Use your definition in (a) to calculate  $f'(2)$ , where  $f(x) = 4 - x^2$ . You receive no credit for using a rule.

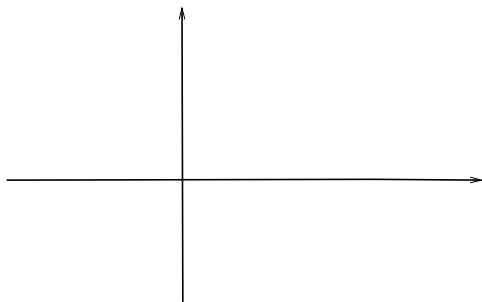
3. (10 pts) (a) State the formal definition of "the function  $f$  is continuous at the point  $x = a$ ":

(b) Using the definition, state why the function defined by  $f(x) = \begin{cases} \frac{x^2+5x-6}{1-x} & \text{if } x \neq 1 \\ -7 & \text{if } x = 1 \end{cases}$  is continuous at  $x = 1$ .

4. (10 pts)

(a) Sketch the graph of  $f(x) = |x - 2| - 2$ .

(b) Determine if  $f(x)$  differentiable at every point and explain your reasoning.



5. (10 pts) Find the equation of the line tangent to the graph of  $y = \frac{2 - x^2}{x^2 + 2}$  at the point  $x = 1$ .

6. (10 pts) The data in the two tables below describes two functions. For each function, decide whether or not it is exponential. If it is, find a formula for it. If it is not then explain why it cannot be exponential.

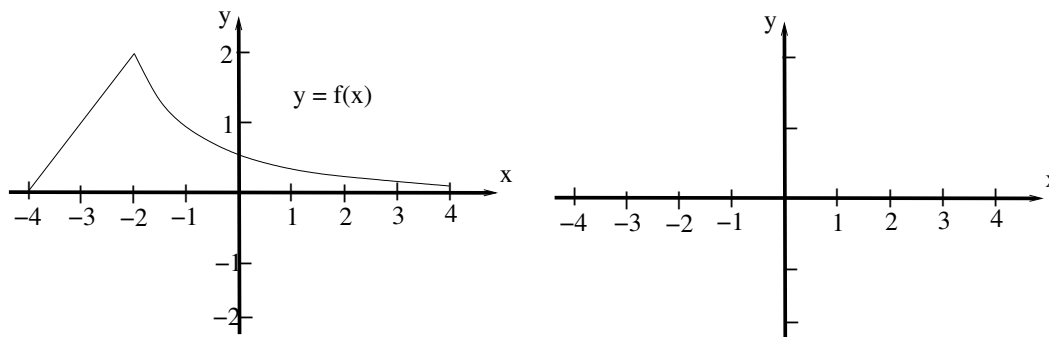
t	f(t)
0	120
1	90
2	67.5
3	50.625
4	37.969
5	28.477

t	g(t)
0	120
1	90
2	60
3	30
4	0
5	-30

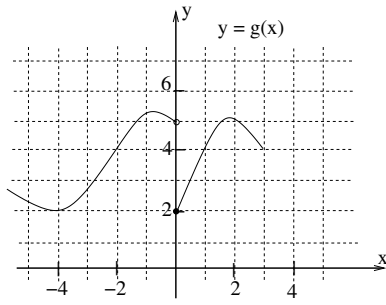
(a)  $f(t)$

(b)  $g(t)$

7. (10 pts) Given the following graph of  $f(x)$ , sketch the graph of the derivative,  $f'(x)$ .



8. (10 pts) Suppose  $f(x) = |x|/x$ , and suppose  $g(x)$  is given by the following graph:



(a) Calculate  $\lim_{x \rightarrow 0^+} f(x)g(x) =$

(b) Calculate  $\lim_{x \rightarrow 0^-} 2f(x) + 3g(x) =$

9. (10 pts)

(a) State the Intermediate Value Theorem.

(b) Use the Theorem to determine whether or not the function  $f(x) = x^5 - 6x - 1$  has a root (zero) between  $x = 1$  and  $x = 2$ . Explain carefully how you used the Intermediate Value Theorem to obtain your answer.

10. (10 pts) Suppose  $V(t)$  is the value of a stock portfolio, where  $t$  is the time in years since January 1, 2000. Give a practical interpretation (using sentences about the portfolio), of the following numbers.

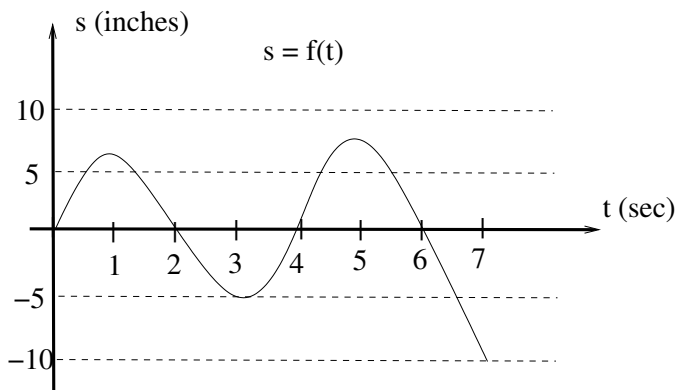
(a)  $V(1.5) = 90,000$

(b)  $V'(1) = 10,000$

(c)  $V'(3)$  is positive and  $V''(3)$  is negative

(d)  $V^{-1}(100,000) = 2.$

11. (10 pts) A marble is rolling back and forth along a rail. Let  $O$  denote a fixed point on the rail, and let  $s = f(t)$  denote the marble's position (in inches) relative to  $O$  at time  $t$  (in seconds).  $s$  is positive to the right of the point  $O$  and negative to the left of  $O$ . The graph of the function  $s = f(t)$  is given below.



- (a) What is the average velocity of the marble over the first 7 seconds?
- b) On which time intervals is the marble rolling to the right?
- c) On which time intervals is the marble rolling to the left?
- d) On which time intervals does the marble exhibit positive acceleration? negative?
- e) On which time intervals is the marble speeding up? slowing down?