Math 113 **Exam 3** Spring 2002

**Name:**_____ Section:_____

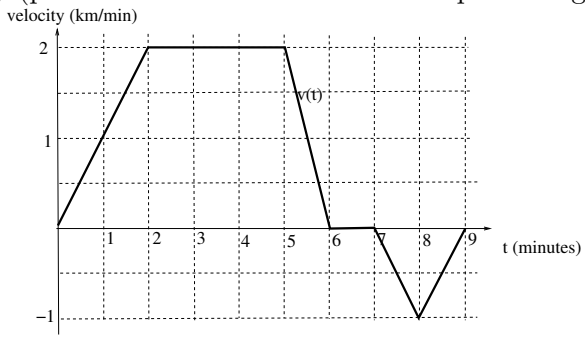
IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

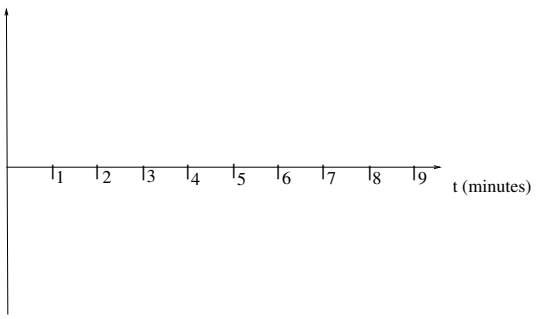
1. (10 pts) A car is going 80 feet per second and the driver puts on the brakes, bringing the car to a stop in 5 seconds. Assume the acceleration (really deceleration) is constant while the brakes are on.

   (a) Determine the velocity at time $t$ and the acceleration.

   (b) How far does the car travel from the time the brakes are applied until it stops?

2. (10 pts) A car is moving along a straight road from $A$ to $B$, starting from $A$ at time $t = 0$. Below is the velocity (positive direction is from $A$ to $B$ plotted against time.




   (a) Sketch a graph of the acceleration of the car against time. Label your axes.

   (b) How many kilometers is the car from $A$ at time $t = 6, 7$, and 9?

   (c) What is the averge velocity over the interval $0 \leq t \leq 9$?

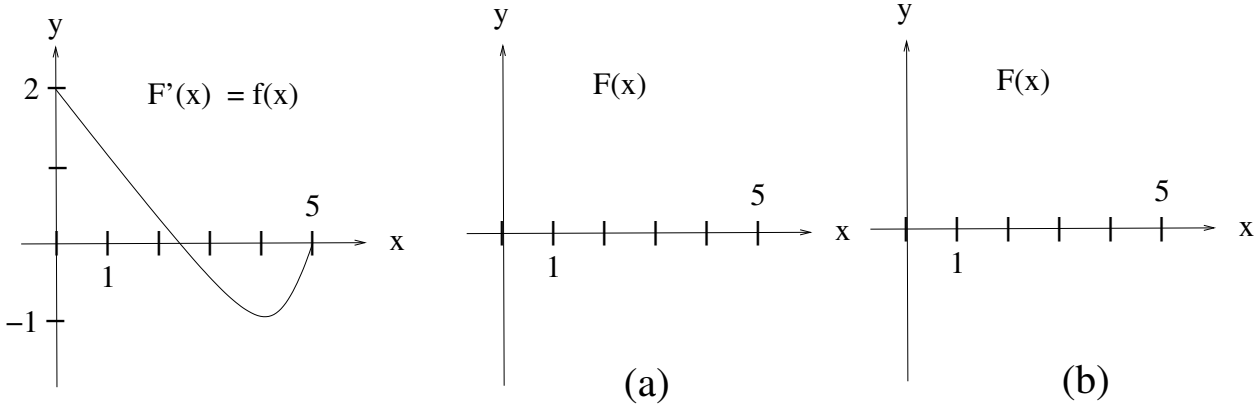
3. (10 pts) For $-1 \le x \le 1$, define $F(x) = \int_{-1}^{x} \sqrt{1-t^2}\, dt$.

(a) Sketch a graph of $y = \sqrt{1-t^2}$ and explain what $F(1)$ represents geometrically.

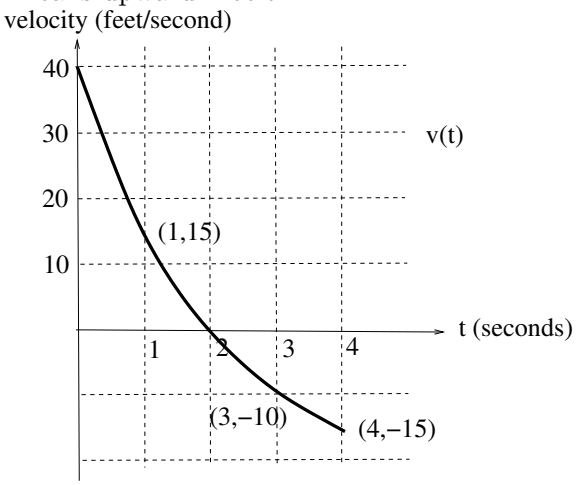
(b) What is the value of $F(-1)$? Of $F(0)$?

(c) Find $F'(x)$.

4. (10 pts) The graph of $f(x)$ is shown below. Sketch the graphs of two functions $F$ where $F'(x)) = f(x)$. For (a) let $F(0) = 0$ and for (b) let $F(0) = 3$. In both cases, identify local maxima, local minima, and inflection points of $F(x)$.



5. (10 pts) Below is the graph of the velocity, in feet per second, of a hat that is thrown up in the air from ground level. Positive velocity means upward motion.




(a) At what time does the hat reach the top of its flight?

(b) Estimate the acceleration of the hat at time $t = 3$ and explain how you arrived at it.

(b) Find an estimate of the height of the hat at time $t = 4$ using a right sum in which the interval $[0, 4]$ is partitioned into 4 intervals. Shade the rectangles and write out the terms in the sum, showing your work.

6. (10 pts) Find $\dfrac{d}{dx}\displaystyle\int_1^{x^3+x} \frac{\sqrt{t^2+3}-2}{t+1}\,dt.$

7. (10 pts) Use the Fundamental Theorem to find the total area of the region bounded between the curve $f(x) = x^3 - 5x^2 + 4x$ and the $x$-axis. Show your work by finding the antiderivative of $f$, determining the appropriate limits, and evaluating the definite integrals involved using the Fundamental Theorem. (HINT: Sketch a graph of $f(x)$ and use the fact that it factors as $f(x) = x^3 - 5x^2 + 4x = x(x-4)(x-1)$.)

8. (10 pts) Suppose you know that $\int_{-1}^{6} f(x)\,dx = 2$ and $\int_{6}^{3} f(x)\,dx = 5$. What is $\int_{-1}^{3} f(x)\,dx$?

9. (10 pts) (a) Find the antiderivative $\displaystyle\int \frac{x^3 - 4x + \sqrt{x}}{x}\, dx$

10. (10 pts) Evaluate the definite integral (exactly!) $\displaystyle\int_{\pi/2}^{0} 5\sin x\, dx$

11. (10 pts) Find the antiderivative $\displaystyle\int e^{2x}\, dx$

12. (10 pts) Use $w$-substitution to evaluate $\int te^{8t}\,dt$.

13. (10 pts) Find the integral $\int x^3(4-x^4)^5\,dx$.