Math 211 **Practice Final Exam**

Disclaimer: As usual, I did not write this final and you'll notice slightly different notation in places. However, I like all these questions and they are good practice for you.

1. Use Euler's Method (with $\Delta t = 0.1$) to approximate $y(0.3)$ where $y(t)$ is the solution to the initial value problem:

$$\frac{dy}{dt} = t^2 - y, \quad y(0) = 1.$$

2. Consider the following system of differential equations that represent the predator-prey model:

$$\begin{cases} \dfrac{dx}{dt} = 2x(1 - \dfrac{x}{2}) - 3xy \\ \dfrac{dy}{dt} = -y + xy \end{cases}$$

(a) Find the equilibrium points and explain their significance in terms of the predator and prey populations.

(b) Is the growth of the prey population limited by any factors other than the number of predators? Explain.

(c) What happens to the predator population if the prey become extinct?

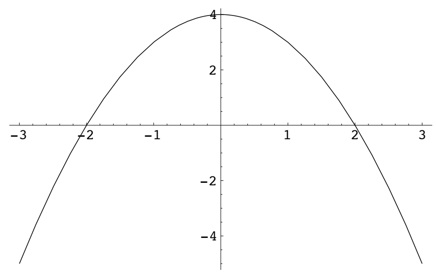
3. Solve the following initial value problem using **TWO** different methods. **One method must be the Method of Laplace Transforms.**

$$\frac{dy}{dt} - 3y = e^{-t}, \quad y(0) = -2$$

(a) Method of Laplace Transforms

(b) Method 2

4. Let $f(y)$ be the function:



(a) Draw the phase line corresponding to the differential equation $\frac{dy}{dt} = f(y)$

(b) Use the phase line to draw an approximate graph of the solution to the initial value problem $\frac{dy}{dt} = f(y), \quad y(0) = -1$

(c) What are the bifurcation values of the differential equation $\frac{dy}{dt} = f(y) + c$. Explain your answer in words, you do not need to draw any pictures or perform any calculations.

5. Solve the following linear system.

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

(a) Write the general solution for this system.

(b) Draw the phase plane for this system. As always, include many different solutions with arrows indicating direction and label the axes.

6. Solve the following linear system.

$$\frac{d}{dt}\begin{bmatrix} \text{x(t)} \\ \text{y(t)} \end{bmatrix} = \begin{bmatrix} \text{-2} & \text{-3} \\ 3 & \text{-2} \end{bmatrix}\begin{bmatrix} \text{x(t)} \\ \text{y(t)} \end{bmatrix}$$

(a) Write the general solution for this system.

(b) Draw the phase plane for this system. As always, include many different solutions with arrows indicating direction and label the axes.

7. Consider a harmonic oscillator (mass-spring system) modeled by the following differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = \cos 2t, \quad y(0) = 1, \quad y'(0) = 0$$

(a) Consider the homogeneous equation (i.e. no outside force acting on the system) Classify the type of the oscillator as undamped, underdamped, crtically damped, or overdamped?

(b) Find the particular solution to the initial value problem.

8. What is the inverse Laplace transform of

$$Y(s) = \frac{s+3}{s^2+9} - \frac{1}{s^3}?$$

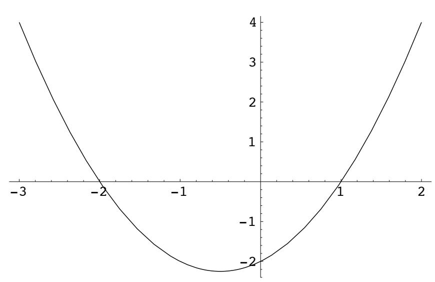
9. Most of the following questions are multiple choice. Circle **ALL** answers that apply.

   (a) What would you draw at the point $(1, 2)$ in the slope field corresponding to the differential equation:

   $$\frac{dy}{dt} = t^2 - y$$

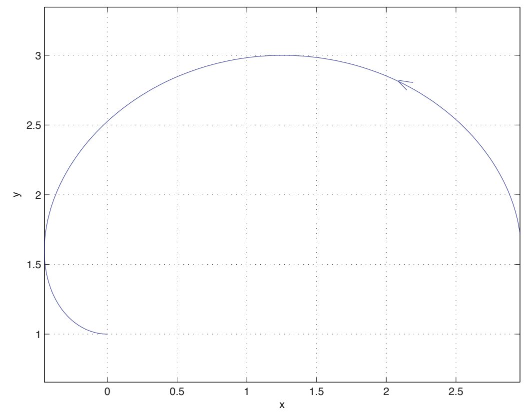
   i. A line segment with slope 2.
   ii. A line segment with slope -1.
   iii. A line segment with slope 3.
   iv. A line segment with slope 0.
   v. None of the above.

   (b) If $y(t)$ is a solution to the initial value problem $\frac{dy}{dt} = f(y), \quad y(0) = -1$, where $f(y)$ is the function graphed below, what is the $\lim_{t\to\infty} y(t)$?



   i. $-\infty$
   ii. -1
   iii. 1
   iv. -2
   v. ∞

(c) Given the phase plane, draw the graph of $x(t)$ on a tx-plane.



(d) If $\frac{d\mathbf{Y}}{dt} = A\mathbf{Y}$ and the eigenvalues of A are $\lambda = -2 \pm i$, which of the following are possible phase portrait(s) for the system?

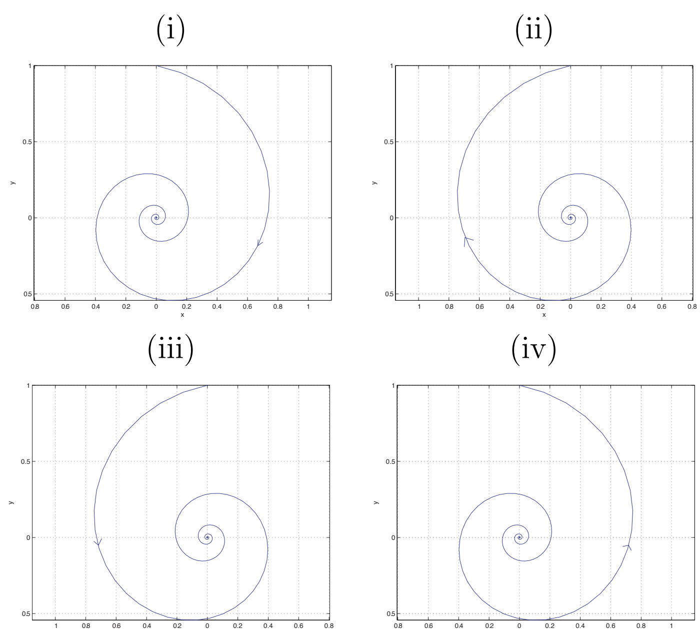


Table of Laplace Transforms

$$\mathscr{L}[e^{at}] = \frac{1}{s-a}$$

$$\mathscr{L}[\sin \omega t] = \frac{\omega}{s^2+\omega^2}$$

$$\mathscr{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathscr{L}[\cos \omega t] = \frac{s}{s^2+\omega^2}$$

$$\mathscr{L}[u_a(t)] = \frac{e^{-as}}{s}$$