Math 210: Practice Final Exam

1. (10 points) Determine whether the planes $4z = 3y - 2x$ and $x + 6y + 4z = 1$ are parallel, perpendicular, or neither.

2. Let $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

   (a) (10 points) Find the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (b) (10 points) Find a unit vector orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

3. (10 points) Find the directional derivative of $f(x, y, z) = ze^{xy^2}$ in the direction of $\mathbf{v} = \langle -1, 2, 3 \rangle$ at the point $(0, 2, 1)$.

4. (10 points) Find the equation of the tangent plane to the surface $z = x^2 \ln y$ at the point $(3, e, 9)$.

5. (10 points) A particle moves on the curve $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} + t\mathbf{k}, \quad t > 0$. Find the velocity, speed and acceleration at any time t.

6. (20 points) Find the amount of work done by the force field $\mathbf{F}$ moving a particle along the curve C, where $\mathbf{F}(x, y, z) = (3y + xz)\mathbf{i} + (x + z^2)\mathbf{j} + (2x^2 - xz^2)\mathbf{k}$ and C is given by the vector equation $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t\mathbf{k}, \ 0 \leq t \leq 1$.

7. (20 points) Find the critical points of $f(x, y) = x^3y^2 - 12x^2 - 8y^2$ and find the local maximum and minimum values of f and any saddlepoints of f.

8. (20 points) Find the volume of the solid E which lies below the graph of $z = x^2 + 3y^2$, and above the region D in the $xy-$plane, where D is bounded by $y = x^2$ and $y = x$.

9. (20 points) Evaluate $\displaystyle\iiint_E 4z \, dV$, where E is the region <u>above the $xy-$plane</u> between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

10. (20 points) Evaluate the line integral $\displaystyle\int_C (ye^x + xy) \, dx + (x^2 + y^5) \, dy$, where C is the square with vertices $(-1, -1), (1, -1), (1, \ 1)$, and $(-1, 1)$, traversed counterclockwise.

11. Consider the line integral $\displaystyle\int_C (x^2 + 3y\cos x) \, dx + (3\sin x + y) \, dy$, where C is any path from $(0, 0)$ to $(1, 2)$.

    (a) (5 points) Show that $\displaystyle\int_C (x^2 + 3y\cos x) \, dx + (3\sin x + y) \, dy$ is independent of path.

    (b) (15 points) Compute $\displaystyle\int_C (x^2 + 3y\cos x) \, dx + (3\sin x + y) \, dy$.

12. (a) (5 points) Find a parametric representation for the surface of the cylinder $x^2 + y^2 = 9$ between the planes $z = 0$ and $z = 4$.

    (b) (15 points) Find the surface integral of $f(x, y, z) = x + z$ over the surface from part (a).

Note: The next two problems were not on the original exam given. So your exam will be a little shorter, but could include problems like these.

13. Use the Divergence Theorem to calulate the outward flux of the vector field

$$\mathbf{v}(x, y, z) = (5x + \sin y \tan z)\,\mathbf{i} + (y^2 + e^{x-\cos z})\,\mathbf{j} + xy^3\,\mathbf{k}$$

across the boundary of the solid T, where T is the part of the solid cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$.

14. Let C be a simple closed curve in the plane $2x + 2y + z = 2$, oriented counterclockwise when viewed from above. Use Stokes' Theorem to show that

$$\oint_C 2y\,dx + 3z\,dy - x\,dz$$

depends only on the area of the region enclosed by C and not on the position or shape of C.